

Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find the Taylor polynomial of degree 3 approximating the function $f(x) = \sqrt[3]{1+x}$ for x near zero.

$$f(x) = \sqrt[3]{1+x} \rightarrow (1+x)^{1/3} \quad f(0) = 1$$

$$f'(x) = \frac{1}{3} (1+x)^{-2/3} \quad f'(0) = \frac{1}{3}$$

$$f''(x) = \frac{-2/9}{(1+x)^{5/3}} \quad f''(0) = \frac{-2}{9}$$

$$f'''(x) = \frac{10}{27} (1+x)^{-8/3} \quad f'''(0) = \frac{10}{27}$$

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$P(x) = 1 + \frac{1}{3}x + \frac{-2/9}{2}x^2 + \frac{10/27}{6}x^3$$

$$P(x)_3 = 1 + \frac{1}{3}x + \frac{-1}{9}x^2 + \frac{5}{61}x^3$$

Great
Job

2. Find the Taylor polynomial of degree 5 for the function $g(x) = \sin x$ centered at $x = 0$.

$f(x) =$	$\sin x$	$f(0) \rightarrow$	0
$f'(x) =$	$\cos x$	\rightarrow	1
$f''(x) =$	$-\sin x$	\rightarrow	0
$f'''(x) =$	$-\cos x$	\rightarrow	-1
$f^{(4)}(x) =$	$\sin x$	\rightarrow	0
$f^{(5)}(x) =$	$\cos x$	\rightarrow	1

Take the derivative to the 5th deg. put in 0 to each derivative + then put those #'s in to find the coefficients in the Taylor Polynomial.

$$= 0 + 1x + \frac{0}{2}x^2 + \frac{-1}{6}x^3 + \frac{0}{24}x^4 + \frac{1}{120}x^5$$

$$= 1x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

Good