Quiz 6a **Calculus 2** **4/2/04**

Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find a solution to the differential equation $\frac{dm}{dt} = 100 - 0.3m$ subject to the initial condition that $m(0) = 400$.

$\frac{dm}{dt} = 100 - 0.3m$

Great

0.1, $\int \frac{1}{100-0.3m} = \int dt$

0.1, $-\frac{1}{0.3} \ln|100-0.3m| = t + c$

0.1, $\ln|100-0.3m| = -0.3t - c$

0.1, $100 - 0.3m = e^{-0.3t-c}$

0.1, $100 - 0.3m = Ae^{-0.3t}$

0.1, $-0.3m = Ae^{-0.3t} - 100$

$\therefore m(t) = \frac{100 - Ae^{-0.3t}}{0.3}$

$m(0) = 400$

$\therefore 400 = \frac{100 - Ae^{-0.3\times 0}}{0.3}$

0.1, $120 = 100 - Ae^{0}$

0.1, $A = 100 - 120$
$= -20$

Hence, the sol^n of the diff. equa^n is

$m(t) = \frac{100}{0.3} + \frac{20}{0.3}e^{-0.3t}$

0.1, $\boxed{m(t) = 333.33 + 66.67e^{-0.3t}}$

2. Lake Superior has a volume of approximately 12.2 thousand km^3, and an outflow rate of roughly 65.2 km^3 per year. Write a differential equation that models the quantity Q of some pollutant in the lake over time.

$\frac{dQ}{dt} = \frac{-65.2}{12200} \cdot Q$

$\frac{Q}{12200}$ is the proportion of the total water that's polluted, and there are 65.2 km^3 like that leaving, hence the negative.