Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find a general solution to the differential equation $y'' + y' - 6y = 0$.

Assume $y = e^{8t}$

$y' = 8e^{8t}$

$y'' = 64e^{8t}$

$e^{8t} (8^2 + 8 - 6) = 0$

$e^{8t} (64 + 8 - 6) = 0$

$e^{8t} (68) = 0$

$S = -3, S = 2$

So $y = ae^{-3t} + be^{2t}$

2. If you know that the differential equation $y'' + 3y' + 2y = 0$ has the general solution $y = ae^{-t} + be^{-2t}$, find a particular solution that satisfies the conditions $y(0) = 0$ and $y'(0) = 1$.

We have,

$y = ae^{-t} + be^{-2t}$

But, $y(0) = 0$

So, $0 = a + b$

So, $a + b = 0$

$a = -b$

Now,

$y' = -ae^{-t} - 2be^{-2t}$

Also, $y'(0) = 1$

So, $1 = -a - 2b$

$1 = -(-b) - 2b$

But, $a = -b$

So, $1 = b - 2b$

$a = -b$

So, $1 = -b$

$b = -1$

$a = 1$

$y = e^{-t} - e^{-2t}$