

Exam 1 Differential Equations 2/6/04

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. **Verify** that $y = e^{2t}$ is a solution to the differential equation $y'' - 5y' + 6y = 0$.

2. **Give examples** of (make clear which is which!):

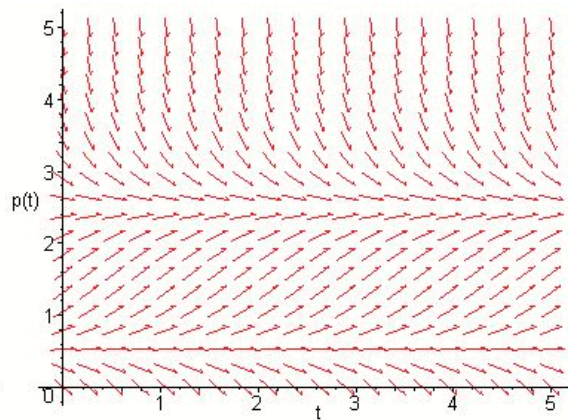
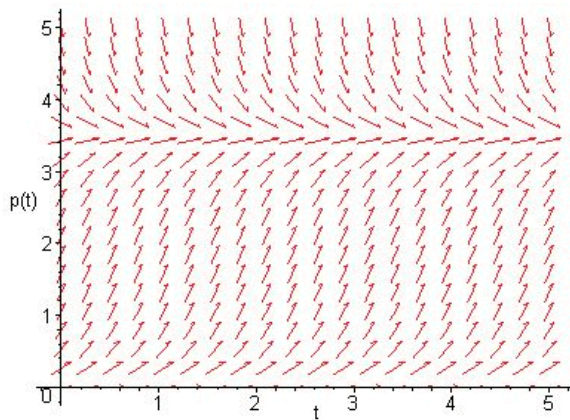
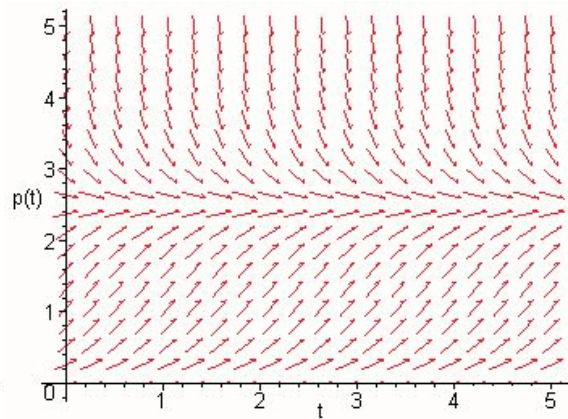
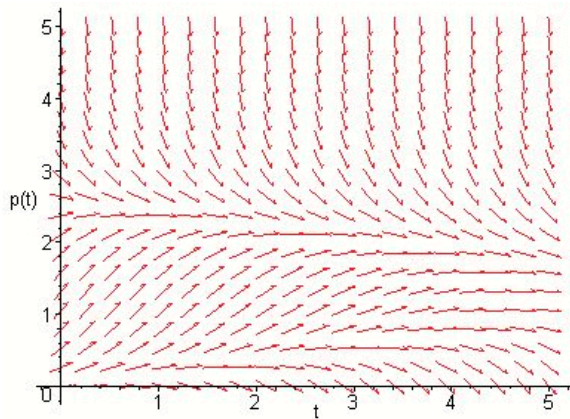
(a) An autonomous differential equation

(b) A non-autonomous differential equation

3. Jon's bedroom (which has a volume of roughly 2000 cubic feet) reached a peak of 45 parts per million of carbon monoxide at 3am, at which point he opened the windows. If 5 cubic feet of clean air (with zero parts per million of carbon monoxide) entered through one window each minute and 5 cubic feet of air from the room exited through the other window each minute, **write a differential equation** for the carbon monoxide level in the room t minutes after 3am.

4. Which of the slope fields pictured below could represent the differential equation

$$\frac{dP}{dt} = -P(P - 2.5)?$$



5. If the differential equation $\frac{dT}{dt} = 0.05(400 - T)$ represents the temperature after t minutes of a toy mouse (which starts out at 70°) that Jon's cat Nemo drops into the oven, **use Euler's method** with $\Delta t = 5$ to estimate the temperature of the mouse ten minutes later when Jon notices that something smells funny.

6. Find the general solution to the differential equation $\frac{dy}{dt} = \frac{t}{t^2y + y}$.

7. Jon's friend Chris went to grad school in an environmental studies program. Chris had a graduate level course where they did remarkably simplistic modeling of populations over time. He got a vague idea there that sometimes you could use differential equations with quadratic terms in them to represent interactions, so he's tried to make a model for the population of elephants in a game preserve in Africa. Since the quadratic term represents interactions, Chris thinks it should have a positive coefficient (more meetings between boy elephants and girl elephants leads to more baby elephants, right?). He's very confused by the slope fields he's getting from his model, and thinks something might be wrong. **Explain** clearly and in simple terms to Chris whether a quadratic function with positive leading coefficient is well suited to modeling population growth of elephants, and why.

8. **Find a particular solution** to the differential equation $\frac{dy}{dt} = e^{2t} + 3y$ subject to the initial condition $y(0) = 3$.

9. **Sketch the bifurcation diagram** for the differential equation $\frac{dy}{dt} = y^6 - 2y^4 + \alpha$. Include direction arrows on the phase lines and make clear the exact α values where bifurcations occur.

10. **Find a general solution** to the differential equation $(t + y) \frac{dy}{dt} = t - y$ [Hint: The substitution $u = t + y$ might be helpful].