Exam 2 Differential Equations 3/5/04

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. **Verify** that $y = e^{2t}$, $v = 2e^{2t}$ is a solution to the system of equations $\begin{aligned} \frac{dv}{dt} &= -6y + 5v \\ \frac{dy}{dt} &= \quad\quad v \end{aligned}$.

2. State the definition of the Laplace transform of a function $y(t)$.

3. Suppose that the populations of rabbits and ferrets are governed by the differential equations $\frac{dR}{dt} = 2R - 1.2RF$ and $\frac{dF}{dt} = -F + 1.2RF$. If the rabbit population begins at 2 and the ferret population begins at 1 (where both populations are measured in thousands), use Euler's method with step size $\Delta t = 0.5$ to find the missing value from the table below (do not round).

t	R	F
0	2	1
.5	2.8	1.7
1	2.744	

4. Find all equilibrium points of the system $\frac{dx}{dt} = 5x\left(1 - \frac{x}{5}\right) - xy$, $\frac{dy}{dt} = 3y\left(1 - \frac{y}{3}\right) - 2xy$.

5. Convert the differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 12y = 0$ to a system of two first order differential equations.

6. Find a solution to the differential equation $y + y' = 3x^2 + 2x$ by assuming there is a second degree polynomial solution.

7. Prove that $L\left[\frac{dy}{dt}\right] = sL[y] - y(0)$.

8. Find a **general** solution to the system $\begin{aligned} \frac{dx}{dt} &= -2x + y \\ \frac{dy}{dt} &= \quad 3y \end{aligned}$.

9. Find a solution to the differential equation $y'' - 4y = 2e^{3t}$ by assuming there is a solution of the form $y = Ae^{3t}$.

10. a) Find a general solution to the differential equation $y'' - 4y = 0$.

b) Find a solution to the differential equation $y'' - 4y = 2e^{3t}$ satisfying the initial condition $y(0) = 5$ [This takes some insight, but think about how your answers to 9. and 10. a) can be combined].