Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Give an example of eigenvalues that would produce a saddle at the origin.

2. If you know that a planar system has eigenvalues \( \lambda_1 = 4 \) and \( \lambda_2 = 0 \), with corresponding eigenvectors \( \mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \) and \( \mathbf{v}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \), write a general solution for the system.
3. Which of the phase planes shown below could correspond to a planar system with eigenvalues \( \lambda_1 = 2 + i \) and \( \lambda_2 = 2 - i \)?
4. Find a **general** solution to the system

\[
\frac{dx}{dt} = -2x + y \\
\frac{dy}{dt} = 3y
\]
5. The planar system has eigenvalues \( \lambda_1 = 2 + 3i \) and \( \lambda_2 = 2 - 3i \), and corresponding eigenvectors \( v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \) and \( v_2 = \begin{pmatrix} 1 \\ i \end{pmatrix} \). Write a general solution to this system in a form not involving imaginary exponents.
6. For which values of the constant $b$ will the system of differential equations have solutions involving sine and cosine?

\[
\frac{dx}{dt} = 2x + by \\
\frac{dy}{dt} = 4x + 6y
\]
7. Find a particular solution to the planar system \( \frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix} \mathbf{Y} \) which satisfies the initial condition \( \mathbf{Y}_0 = (1,0) \).
8. Suppose that the planar system \( \frac{dY}{dt} = \begin{pmatrix} -0.2 & 0.1 \\ 0.0 & -0.1 \end{pmatrix} Y \) represents the populations of two species of fish, creatively labeled as “species X” and “species Y”, where \( x(t) \) actually gives the number of fish of species X in a pond above or below the natural equilibrium population and \( y(t) \) gives the number of fish of species Y present in the pond.

The eigenvalues of the system are \( \lambda_1 = -0.2 \) and \( \lambda_2 = -0.1 \), with associated eigenvectors \( V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)

and \( V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \).

(a) Does the presence of fish of species Y benefit or hurt the population of species X?

(b) If all the fish of species Y are exterminated, what will happen to species X?

(c) If a small number of fish of species Y are introduced, with species X beginning at equilibrium, what will happen?
9. Convert the second-order differential equation \( ay'' + by' + cy = 0 \) to a system of first-order equations and find the equation in \( \lambda \) whose roots are the eigenvalues of the system.
10. Jon is trying to make up a problem for a differential equations test. He wants a planar system with eigenvalues 2 and 3, with respective eigenvectors \( \begin{pmatrix} 1 \\ 3 \end{pmatrix} \) and \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). He knows that you can make a matrix with desired eigenvalues by putting the eigenvalues you want as entries on the main diagonal of the coefficient matrix with a zero in one of the other spots, but what coefficient matrix does he need in order to have his desired eigenvectors?