Exam 1a Calc 2 2/4/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \sin t \, dt$.

2. Suppose that you have an income source that pays you at a rate of $1000 per year right now, but will drop off steadily to $0 per year over the next 20 years. How much money will you receive from this source in total over the next 20 years?

3. When the value of $\int_0^{0.2} e^{(-x^2)}\,dx$ is approximated, *Maple* says that $L_5 \approx 0.1981$, $R_5 \approx 0.1965$, and $M_5 \approx 0.1974$. What will T_5 and S_5 be?

4. Find the general antiderivative of $f(t) = \dfrac{t}{1+5t^2}$.

5. Evaluate $\int xe^{x}\,dx$.

6. Evaluate $\int_3^{\infty} \frac{1}{x^2} dx$.

7. Biff is a calculus student at Anonymous State University, and he's having some trouble with antiderivatives. He says "Dude, this sucks so bad! We've gotta take this calculus test on the computers, and it's totally kicking my butt. It's like, five antiderivative problems, and you've gotta get at least four right to pass, but it's multiple choice from five answers, so the first days when I tried just guessing most of 'em it went really bed. But you can only try it once each day, and I failed five times now, so I gotta get it tomorrow or I lose like twenty points every day. So the thing is, I heard these smart guys in the class talking, and they were laughing about how easy it was. They said something about just working the answers backwards and it only took a couple minutes, but I guess that's something fancy that we haven't learned in class yet. Do you know what they meant about backwards?"

Explain clearly to Biff how to tell which answer is correct, given an indefinite integral and a list of possible answers.

8. Derive the formula $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\left(\ln|x-a| - \ln|x-b|\right) + C$.

9. The integral $\int_0^R \frac{R}{\sqrt{R^2 - x^2}}\,dx$ turns out to be important (I'll tell you why in the next chapter). Evaluate it.

10. a) Find the area below $y = \sqrt{9 - x}$ but above the x axis and to the right of the y axis.

b) Find the value of b for which the area below $y = \sqrt{b - x}$ but above the x axis and to the right of the y axis is equal to 9.

Extra Credit (5 points possible):

Show that the area of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by πab.