Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \sin t \, dt$.

$\boxed{-\cos t + C}$

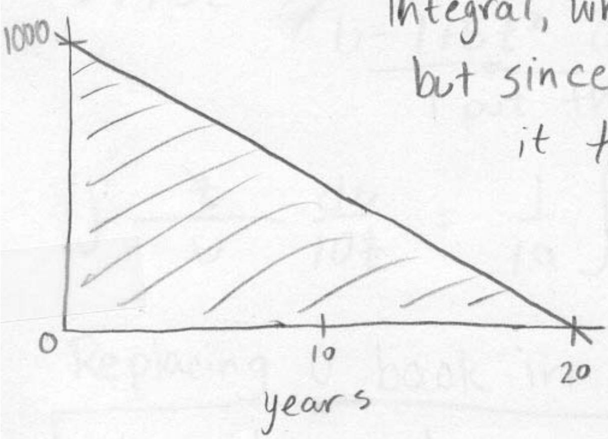
Great

Justification

we know its right b/c if we took the derivative of $-\cos t$ we get $\sin t$

2. Suppose that you have an income source that pays you at a rate of $1000 per year right now, but will drop off steadily to $0 per year over the next 20 years. How much money will you receive from this source in total over the next 20 years?

P=$1000 If I had an equation I would take the integral, which means area under the curve, but since I have this graph, I'll just calculate it that way.



$(20 * 1000)/2 = \boxed{\$10,000}$

Slick!

3. When the value of $\int_0^{0.2} e^{(-x^2)}dx$ is approximated, *Maple* says that $L_5 \approx 0.1981$, $R_5 \approx 0.1965$, and $M_5 \approx 0.1974$. What will T_5 and S_5 be?

T_5 is the average of $L_5 + R_5 = \frac{.1981 + .1965}{2} = .1973$

S_5 stands for Simpson's Rule which is $\frac{M + M + T}{3}$

$\frac{\overset{T}{.1973} + \overset{M}{.1974} + \overset{M}{.1974}}{3} = .19737$ Simpson's rule gets us closest to the exact answer

Excellent

4. Find the general antiderivative of $f(t) = \frac{t}{1+5t^2} \cdot dt$ let $u = 1+5t^2$ $\frac{du}{dt} = 10t$, $dt = \frac{du}{10t}$

$$\int \frac{\cancel{t}}{u} \frac{du}{10\cancel{t}} = \frac{1}{10}\int \frac{1}{u}\,du = \frac{1}{10}\ln|u| + C$$

$$= \frac{1}{10}\ln|1+5t^2| + C$$

Wonderful

I used u substitution to find the answer. To check I can take the deriv. of my answer

$\ln|1+5t^2| = \frac{t}{1+5t^2} \cdot 10t$, the ten cancells out by adding a $\frac{1}{10}$ and the t moves to the top - and add C.

5. Evaluate $\int x e^x \, dx$.

Integration by parts

$u = x$ $\quad v = e^x$ $\quad uv - \int u'v \, dx$

$u' = 1$ $\quad v' = e^x$

$= xe^x - \int e^x \, dx$

$= xe^x - e^x + C$

I used int. by parts and the formula

To check my answer, I find the deriv of my answer. using the product rule:

$\cancel{e^x} + xe^x \cancel{- e^x}$ gives me my original problem.

Then I added C.

Well done

6. Evaluate $\int_3^\infty \frac{1}{x^2}\,dx$.

$$\int_3^\infty \frac{1}{x^2}\,dx$$

$$= \lim_{b\to\infty} \int_3^b x^{-2}\,dx$$

$$= \lim_{b\to\infty} \left[\frac{x^{-2+1}}{-2+1}\right]_3^b$$

$$= \lim_{b\to\infty} \left[\frac{x^{-1}}{-1}\right]_3^b$$

$$= \lim_{b\to\infty} -\left[\frac{1}{x}\right]_3^b$$

$$= \lim_{b\to\infty} -\left[\frac{1}{b} - \frac{1}{3}\right]$$

$$= \lim -\left[\frac{1}{\infty} - \frac{1}{3}\right]$$

$$= -\left[0 - \frac{1}{3}\right]$$

$$= \frac{1}{3}$$ converges

Excellent

7. Biff is a calculus student at Anonymous State University, and he's having some trouble with antiderivatives. He says "Dude, this sucks so bad! We've gotta take this calculus test on the computers, and it's totally kicking my butt. It's like, five antiderivative problems, and you've gotta get at least four right to pass, but it's multiple choice from five answers, so the first days when I tried just guessing most of 'em it went really bed. But you can only try it once each day, and I failed five times now, so I gotta get it tomorrow or I lose like twenty points every day. So the thing is, I heard these smart guys in the class talking, and they were laughing about how easy it was. They said something about just working the answers backwards and it only took a couple minutes, but I guess that's something fancy that we haven't learned in class yet. Do you know what they meant about backwards?"

Explain clearly to Biff how to tell which answer is correct, given an indefinite integral and a list of possible answers.

Hi Biff! First it is important for you to understand that antiderivatives are simply derivatives done backwards. ~~For ex~~ If you take a function say $y = x^2 + c$ find its derivative you'll get $\frac{dy}{dx} = 2x$ or $dy = 2x\,dx$. Now when you take the antiderivative of $2x\,dx$ we get $\int dy = \int 2x\,dx = 2\left[\frac{x^2}{2}\right] + c$

$= x^2 + c$.

Look Biff, we got the same $x^2 + c$ after we took its derivative & the antiderivative of the subsequent function. Your multiple choice test would have been a lot easier if you simply took the derivatives of the possible answers & see which one matches the question, i.e. the indefinite integral. I hope you now understand.

Excellent

8. Derive the formula $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\left(\ln|x-a| - \ln|x-b|\right) + C$.

To derive this we would use partial fractions

I wish $\int \frac{1}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} =$

$A(x-b) + B(x-a) = 1$ if $x = a$ then $A =$

$A(a-b) + B(a-a) = 1 \quad A(a-b) = 1 \quad A = \frac{1}{a-b}$

if $x = b$ the $A(b-b) + B(b-a) = 0 + B(b-a) = 1$

$B = \frac{1}{b-a} \qquad \frac{\frac{1}{a-b}}{(x-a)} + \frac{\frac{1}{b-a}}{(x-b)}$ I want this to equal $a-b$ so I just take the negative =

$\frac{1}{a-b} \int \frac{1}{(x-a)} - \frac{1}{(x-b)} =$

this can pull out because it's a constant

$$\boxed{\frac{1}{a-b} \cdot \left(\ln|x-a| - \ln|x-b|\right) + C}$$

we would also take the derivative of this part, but it just equals one

so $\int \frac{1}{x-a} = \ln(x-a)$

Excellent

10

Improper!

9. The integral $\int_0^R \frac{R}{\sqrt{R^2 - x^2}}\,dx$ turns out to be important (I'll tell you why in the next chapter).

Evaluate it.

Trig. Sub.!

let $x = R\sin\theta$

$\frac{dx}{d\theta} = R\cos\theta$

$dx = R\cos\theta\, d\theta$

$\frac{x}{R} = \sin\theta$

$\arcsin\left(\frac{x}{R}\right) = \theta$

$$\int_0^R \frac{R}{\sqrt{R^2 - x^2}}\,dx = \lim_{b\to R} \int_0^b \frac{R}{\sqrt{R^2 - x^2}}\,dx$$

$$= \lim_{b\to R} \int_{x=0}^{x=b} \frac{R}{\sqrt{R^2 - (R\sin\theta)^2}}\,dx$$

$$= \lim_{b\to R} \int_{x=0}^{x=b} \frac{\cancel{R}}{\cancel{R}\sqrt{1 - \sin^2\theta}}\,dx$$

$$= \lim_{b\to R} \int_{x=0}^{x=b} \frac{1}{\sqrt{\cos^2\theta}} \cdot R\cos\theta\, d\theta$$

$$= \lim_{b\to R} R\cdot\theta \Big|_{x=0}^{x=b}$$

$$= \lim_{b\to R} R\cdot\arcsin\left(\frac{x}{R}\right) \Big|_0^b$$

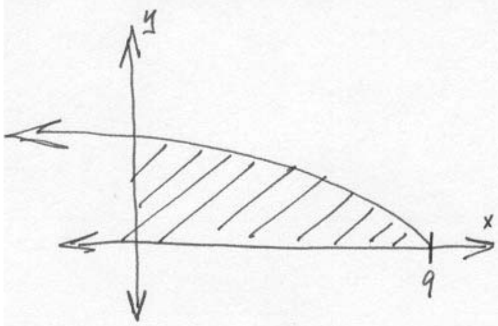
$$= \lim_{b\to R} \left(R\cdot\arcsin\left(\frac{b}{R}\right) - R\cdot\arcsin 0 \right)$$

$$= R\arcsin 1$$

$$= R\cdot\frac{\pi}{2}$$

10. a) Find the area below $y = \sqrt{9-x}$ but above the x axis and to the right of the y axis.

b) Find the value of b for which the area below $y = \sqrt{b-x}$ but above the x axis and to the right of the y axis is equal to 9.



a) $\displaystyle\int_0^9 \sqrt{9-x}\,dx = \int_{x=0}^{x=9} u^{1/2}\cdot -du$

let $u = 9-x$

$\frac{du}{dx} = -1$

$du = -dx$

$= -\frac{2}{3}u^{3/2}\Big|_{x=0}^{x=9}$

$= -\frac{2}{3}(9-x)^{3/2}\Big|_0^9$

$= -\frac{2}{3}\cdot 0 - -\frac{2}{3}\cdot 9^{3/2}$

$= 18$

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b) $\displaystyle\int_0^b \sqrt{b-x}\,dx = \int_{x=0}^{x=b} u^{1/2}\cdot -du$

$= -\frac{2}{3}u^{3/2}\Big|_{x=0}^{x=b}$

$= -\frac{2}{3}(b-x)^{3/2}\Big|_0^b$

$= -\frac{2}{3}\cdot 0 - \frac{-2}{3}\cdot b^{3/2}$

$= \frac{2}{3}b^{3/2}$ ← This gives the area...

So if we want the area to be 9, then set $\frac{2}{3}b^{3/2} = 9$ and solve...

$b^{3/2} = \frac{27}{2}$

$b = \left(\frac{27}{2}\right)^{2/3}$

$b = \frac{9}{\sqrt[3]{4}}$