Exam 3b Calc 2 4/1/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find, to the nearest thousandth, the first 4 partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

2. Find a 6th degree Taylor polynomial for the function f(x) = $e^{\left(-x^2\right)}$.

3. Show that $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

4. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{3^n}$.

5. Use a power series of degree at least 8 to approximate the value of $\int_0^{0.5} \frac{1}{1+x^6}\,dx$ to 8 decimal places.

6. Biff is a calculus student at Anonymous State University, and he's having some trouble with series. Biff says "Man, this convergence stuff is kicking my ass. We just had our test over it and I guess everybody failed, 'cause the professor said we get to take it over again next week to make our grades better. So one of the questions was about whether approximations from Taylor polynomials are always accurate if you use a high enough degree polynomial. I picked the answer that said 'yeah, as long as you don't have an arithmetic mistake', 'cause that's where I screw up a lot. But the machine scored it wrong, which I think is crap, because obviously it's true for me, right?"

Explain clearly to Biff why his answer is actually correct or not, and what sort of answer his professor likely counts as correct.

7. a) Find the sum of the series $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\ldots.$

b) For what value of b is the sum of the series $\frac{1}{b}+\frac{1}{b^2}+\frac{1}{b^3}+\frac{1}{b^4}+\ldots$ equal to ½?

8. A trig identity sometimes used in integration problems says that $2 \sin x \cos x = \sin 2x$. Verify that the 3^{rd} degree Taylor polynomials for both sides of this identity match.

9. Suppose that $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Define a new series $\sum_{n=1}^{\infty} b_n$ by letting $b_n = \begin{cases} a_n & \text{if n is odd} \\ 2a_n & \text{if n is even} \end{cases}$. Does $\sum_{n=1}^{\infty} b_n$ converge?

10. Determine whether $x = -1/a$ is in the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(ax)^n}{n^2}$.

Extra Credit (5 points possible):

If f(x) has Taylor series $\sum_{n=0}^{\infty} a_n x^n$ with radius of convergence r, what is the Taylor series for an antiderivative of f(x) and what can you say about its radius of convergence?