

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the Taylor polynomial of degree 3 for the function $f(x) = \sqrt{1+x}$ near 0.

$(1+x)^{1/2}$

$$\begin{aligned} f(x) &= \sqrt{1+x} & f(0) &= 1 \\ f'(x) &= \frac{1}{2}(1+x)^{-1/2} & f'(0) &= \frac{1}{2} \\ f''(x) &= -\frac{1}{4}(1+x)^{-3/2} & f''(0) &= -\frac{1}{4} \\ f'''(x) &= \frac{3}{8}(1+x)^{-5/2} & f'''(0) &= \frac{3}{8} \end{aligned}$$

formula

$$f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} \dots$$

$$1 + \frac{1}{2}x - \frac{1x^2}{2 \cdot 4} + \frac{3x^3}{8 \cdot 3!}$$

$$1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{x^3}{16}$$

Great

2. Find the first 4 terms of the Taylor series for $g(x) = \sin x$ about the point $x = -\pi/4$.

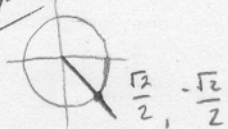
$g(x) = \sin x$	$g'(-\pi/4) = -\frac{\sqrt{2}}{2}$
$g'(x) = \cos x$	$g''(-\pi/4) = \frac{\sqrt{2}}{2}$
$g''(x) = -\sin x$	$g'''(\pi/4) = \frac{\sqrt{2}}{2}$
$g'''(x) = -\cos x$	$g^{(4)}(\pi/4) = -\frac{\sqrt{2}}{2}$
$g^{(4)}(x) = \sin x$	$g^4(-\pi/4) = -\frac{\sqrt{2}}{2}$

formula = $f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} \dots$

$$-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}(x+\pi/4)}{2} + \frac{\sqrt{2}(x+\pi/4)^2}{2 \cdot 2!} + \frac{\sqrt{2}(x+\pi/4)^3}{2 \cdot 3!}$$

$$-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}(x+\pi/4)}{2} + \frac{\sqrt{2}(x+\pi/4)^2}{4} + \frac{\sqrt{2}(x+\pi/4)^3}{12}$$

Excellent!



* The reason this is positive is b/c we had a NEG. $\pi/4$.
Yes!