Exam 1 Foundations of Advanced Mathematics 2/11/2005

Each problem is worth 10 points. Show appropriate justification for full credit. Don't panic.

1. State the definition of an odd integer.

2. Let Λ be some indexing set. State the definition of $\bigcap_{\alpha\in\Lambda} B_\alpha$.

3. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5\}$, and $C = \{2, 3, 4\}$.

a) Compute $A \cap B$.

b) Compute $\mathcal{P}(B)$, the power set of the set B.

c) Compute $A \cup (B \setminus C)$

4. Construct a truth table for the statement $A \Rightarrow (B \vee C)$.

5. Prove that the sum of an odd integer and an even integer is an odd integer.

6. Prove that the product of any three consecutive integers is divisible by 6.

7. Biff is a student taking a math class at Anonymous State University, and he's having some trouble. Biff says "Dude, they just make this math stuff so confusing. So there's this thing in our math book, it says that there's this thing called the twin prime conjection. It says, like, that there's infinitely many times that there's prime numbers just two apart, like how both 11 and 13 are both of 'em prime. So I looked at it for a while, and figured out that it isn't true, 'cause like with 101 and 103, they're both prime. But with 1001 and 1003, you might think they're both prime, but they're not, 'cause 1003 has a 7 in it. So I don't know why the book makes it seem so complicated, when all you gotta do is try factoring a few numbers and you find some."

Explain clearly to Biff how what he says either does or does not refute or confirm the Twin Prime Conjecture (which claims, as Biff mentions, that there are infinitely many pairs of prime numbers with each pair consisting of two numbers which are two units apart from one another).

8. Let A, B, and C be sets. Prove that $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$.

9. Let A, B, and C be sets. Prove or give a counterexample to the statement:

$$\text{If } A \subseteq B, \text{ then } A \cap C \subseteq B \cap C.$$

10. Let A be a set of real numbers. Define an element g of A to be the **greatest element** in A if for any $a \in$ A, $g \geq a$. Prove that the greatest element in A is unique.

Extra Credit (5 points possible):

a) Let Λ be an indexing set with $\{C_\alpha\}_{\alpha\in\Lambda}$ a collection of sets, and suppose that $\bigcup_{\alpha\in\Lambda} C_\alpha = \varnothing$. What can you conclude about C_α or Λ?

b) Let Λ be an indexing set with $\{C_\alpha\}_{\alpha\in\Lambda}$ a collection of sets, and suppose that $\bigcap_{\alpha\in\Lambda} C_\alpha = \varnothing$. What can you conclude about C_α or Λ?