

Problem Set 5 Foundations Due 3/1/2005

Each problem is worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Consider the formula $1 + 2 + 3 + \dots + n = \frac{n^2 + n + 1}{2}$.

- a) Write the formula in sigma notation.
- b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.
- c) Explain why mathematical induction does **not** prove that this formula is true for all $n \in \mathbb{N}$.

2. Consider the relation $(a, b) \sim (c, d)$ iff $ac = bd$. Is this relation:

- a) reflexive?
- b) symmetric?
- c) antisymmetric?
- d) transitive?
- e) a total ordering of $\mathbb{R} \times \mathbb{R}$?

3. Consider the relation $(a, b) \approx (c, d)$ iff $a < c$ or $(a = c$ and $b \leq d)$. Is this relation:

- a) reflexive?
- b) symmetric?
- c) antisymmetric?
- d) transitive?
- e) a total ordering of $\mathbb{R} \times \mathbb{R}$?