Exam 1   Calc 1   2/10/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of the derivative of the function f(x) at the point $x = a$.

Use the graph of f(x) at the bottom of the page for problems 2 and 3:

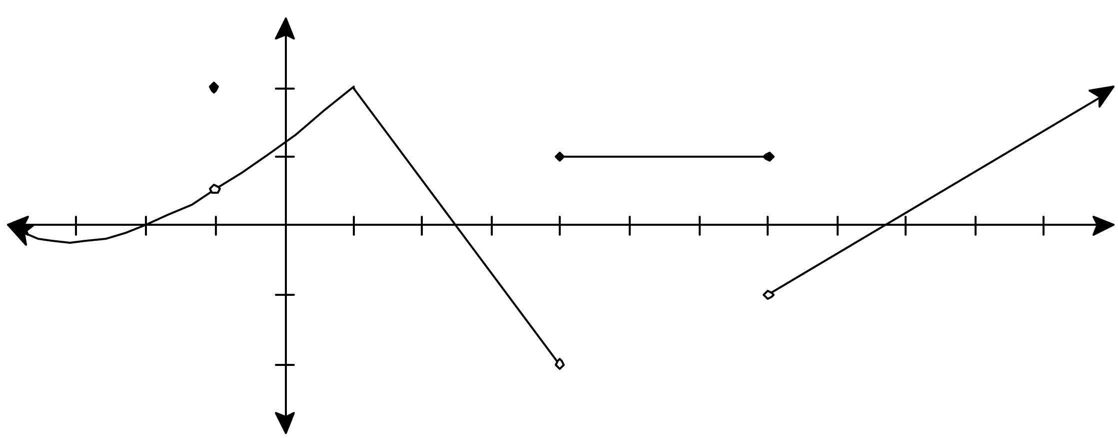
a) What is f(7)?

b) What is $\lim_{x\to 7^+} f(x)$?

c) What is $\lim_{x\to 7} f(x)$?

3. a) For which value(s) of x is f(x) not continuous? Why?

b) For which value(s) of x is f(x) not differentiable? Why?



4. Let $f(x) = 2^x$. Estimate the value of $f'(0)$ by using the definition of the derivative and taking successively smaller values of h.

5. Evaluate $\lim_{x \to -7} \frac{\frac{1}{7} + \frac{1}{x}}{7 + x}$ exactly.

6. If $f(x) = \sqrt{x+1}$ Use the definition of the derivative to find $f'(2)$.

7. Evaluate the limit $\lim_{x\to 1}\frac{4-3x}{x^2+2}$ and justify each step by indicating the appropriate limit law(s) from the list below.

Algebraic Limit Properties

Let c be a constant. Then as long as $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist,

Constant Rule for Limits:	$\lim_{x\to a} c = c$
Rule X for Limits:	$\lim_{x\to a} x = a$
Sum Rule for Limits:	$\lim_{x\to a}\left[f(x)+g(x)\right] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
Difference Rule for Limits:	$\lim_{x\to a}\left[f(x)-g(x)\right] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
Constant Multiple Rule for Limits:	$\lim_{x\to a}\left[c\cdot f(x)\right] = c\cdot\lim_{x\to a} f(x)$
Product Rule for Limits:	$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$
Quotient Rule for Limits:	$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ as long as $\lim_{x\to a} g(x)\neq 0$.

Power Rule for Limits: $$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, we had this test, and it was so messed up. Everything's multiple guess, of course, because there's like a million people in the class, but that means if you get something wrong you get no credit at all, and they won't tell you how to do it, they just put up the right answers afterwards so you learn *nothing*. So there was this question about, like, estimating the slope of the tangie-thing for $1/x$ where $x = -1$, right? So I did it like I remembered from class, where I did the slope from (-1,-1) to the point where x is zero, 'cause it's easy to plug in zero, right? So it came out undefined, so I marked that answer, but they said it was wrong. I have no idea why!"

Explain to Bunny, as clearly as possible, either what issues keep her answer from being correct, or why her answer is in fact right so she can go argue with the professor.

9. Let $f(x) = \dfrac{1}{x^2}$. Use the definition of the derivative to find $f'(a)$.

10. Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+ax+6}-\sqrt{x^2+3x+1}\right)$, where a is some constant.

Extra Credit (5 points possible):
Suppose that $f(x) = x \cdot g(x)$ for some differentiable function $g(x)$. Use the definition of the derivative to find $f'(a)$.