

Exam 2 Calc 1 3/2/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State and prove the Sum Rule for derivatives.

2. Find an equation for the line tangent to the function $f(x) = x^3 - \ln x$ at the point where $x = 1$.

3. Show that the derivative of $\operatorname{csch} x$ is $-\operatorname{csch} x \operatorname{coth} x$.

4. A busload full of clowns in the northbound lane of a divided highway passes a busload full of mimes in the southbound lane at exactly 2pm. If the clowns are traveling at 110 feet per second and the mimes are traveling 130 feet per second, and the northbound and southbound lanes are parallel and 18 feet apart, at what rate is the distance between the clowns and the mimes changing a tenth of a second later, at the sublime moment when one of the mimes glimpses one of the clowns through the back window and falls forever in love?

5. Show that $(\tan^{-1} x)' = \frac{1}{1+x^2}$.

6. a) Suppose $f(x)$ is a differentiable function and $F(x) = \sqrt{f(x)}$. Express $F'(x)$ in terms of $f(x)$ and $f'(x)$.

b) Suppose $g(x)$ is a differentiable function and $G(x) = \ln(g(x))$. Express $G'(x)$ in terms of $g(x)$ and $g'(x)$.

7. State and prove the Quotient Rule for derivatives.

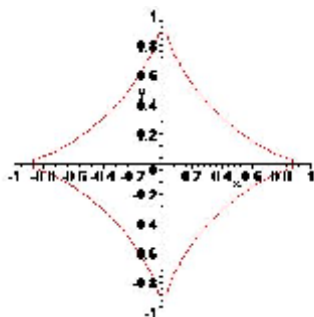
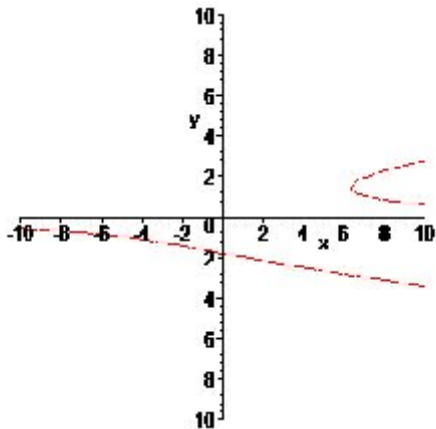
8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dang, this calculus stuff is kicking my ass. I liked it when it was just formulas and stuff, but now there's all this stuff where it's not just working it out. The professor keeps talking about reasoning, like that's a good thing or something. So I guess he was trying to convince us or something, and he said this thing about if you know the derivative of a function, then you also know the derivative of its inverse. It sounded like he thought it was important, but I couldn't figure out what the heck he meant."

Explain clearly to Biff how knowing the derivative of $f(x)$ tells us something about the derivative of $f^{-1}(x)$.

9. Find a second-degree polynomial (so a function of the form $f(x) = ax^2 + bx + c$) for which $f(0) = 5$, $f'(0) = -1$, and $f''(0) = 2$.

10. The graph of the curve with equation $y^3 - xy = -6$ is shown below.

- Find the slope of the line tangent to this curve.
- The graph has an “upper loop”. Find the exact coordinates of the leftmost point on it.



Extra Credit (5 points possible): [Stewart 5th, p. 273] Show that the length of the portion of any tangent line to the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ cut off by the coordinate axes is constant.