Exam 4 Practice Calc 1 2/20/2006

Each problem is worth 0 points. For full credit provide complete justification for your answers.

1. Suppose that the following table of values gives a car's speed as it accelerates after a light turns green, then brakes to a stop at the next intersection. Based on this data, give an upper approximation for the distance the car traveled between the two intersections.

t (seconds)	0	5	10	15
speed (feet/second)	5	40	50	10

2. Use a right-hand sum with three equal subintervals to approximate the value of $\int_0^9 \sqrt{x}\,dx$.

3. Evaluate $\int_0^9 \sqrt{x}\,dx$ exactly.

4. Let $F(x) = \int_0^x \cos\theta \, d\theta$.

   a. Evaluate $F(\pi/2)$.

   b. Evaluate $F(2\pi)$.

5. Let $G(x) = \int_0^x \sin\left(t^2\right) dt$. What is $G'(x)$?

6. Find $\int \frac{x}{1+x^2}\,dx$.

7. Find the area bounded between $y = x^3 - 8x$ and $y = -x^3$.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, I love these multiple choice tests! We had this one in calculus, and I was pretty much just guessing on some of the stuff, but it's so cool because you can totally eliminate some of the answers even if you've got no clue how to do the problem. Like this one, there was one of them integrals, and one of the answers was zero. So like normally they give the area, right, except they give the negative area if it's down below or whatever, but there's no way an area can be zero, right? So I just guessed from all the answers that aren't zero, and my odds are way better, right?"

   Explain clearly to Biff whether this reasoning is valid or not.

9. Write an expression using sigma notation to approximate the area under $f(x) = x^2$ but above the x-axis between $x = a$ and $x = b$ using n subdivisions.

10. Show that $\int_{-\pi/6}^{\pi/6} \sin x \cos^n x \, dx$ is always zero for any positive integer n.

Extra Credit (5 points possible): Let $[\![x]\!]$ denote the greatest integer function, which returns the greatest integer less than or equal to its input (so $[\![3.2]\!] = 3$ and $[\![5]\!] = 5$, for instance). Evaluate $\int_0^n [\![x]\!] \, dx$, where n is a positive integer.