$$A = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix} \qquad (-\lambda)(-\lambda) - (-1)(2) = 0$$

$$\lambda^2 + 2 = 0$$

$$\lambda = \pm \sqrt{2}\, i$$

If $\lambda = \sqrt{2}\, i$:

$$\begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \sqrt{2}\, i \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$-y_1 = \sqrt{2}\, i\, x_1 \qquad \begin{pmatrix} 1 \\ -\sqrt{2}\, i \end{pmatrix} \text{ or } \begin{pmatrix} i \\ \sqrt{2} \end{pmatrix}$$

$$2x_1 = \sqrt{2}\, i\, y_1$$

So one solution is:

$$\hat{Y} = e^{\sqrt{2} i t} \begin{pmatrix} i \\ \sqrt{2} \end{pmatrix} = (\cos \sqrt{2} t + i \sin \sqrt{2} t) \begin{pmatrix} i \\ \sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} i \cos \sqrt{2} t - \sin \sqrt{2} t \\ \sqrt{2} \cos \sqrt{2} t + i \sqrt{2} \sin \sqrt{2} t \end{pmatrix}$$

$$= \begin{pmatrix} -\sin \sqrt{2} t \\ \sqrt{2} \cos \sqrt{2} t \end{pmatrix} + i \begin{pmatrix} \cos \sqrt{2} t \\ \sqrt{2} \sin \sqrt{2} t \end{pmatrix}$$

And a general solution is:

$$\hat{Y} = k_1 \begin{pmatrix} -\sin \sqrt{2} t \\ \sqrt{2} \cos \sqrt{2} t \end{pmatrix} + k_2 \begin{pmatrix} \cos \sqrt{2} t \\ \sqrt{2} \sin \sqrt{2} t \end{pmatrix}$$

So a particular solution satisfying $(1, 0)$ is:

$$\hat{Y} = \begin{pmatrix} \cos \sqrt{2} t \\ \sqrt{2} \sin \sqrt{2} t \end{pmatrix}$$

$A = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$ So solve $(2-\lambda)(0-\lambda) - (-1)(2) = 0$

$$-2\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= 1 \pm i$$

Taking $\lambda_1 = 1+i$:

$$\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = (1+i) \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$2x_1 - y_1 = (1+i)\,x_1 \qquad\qquad v_1 = \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

$$2x_1 = (1+i)\,y_1$$

So one solution is:

$$\hat{Y} = e^{(1+i)t} \begin{pmatrix} 1+i \\ 2 \end{pmatrix} = e^t \cdot e^{it} \begin{pmatrix} 1+i \\ 2 \end{pmatrix} = e^t(\cos t + i \sin t) \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

$$\hat{Y} = e^t \begin{pmatrix} \cos t + i\cos t + i \sin t - \sin t \\ 2\cos t + i\,2\sin t \end{pmatrix} = e^t \begin{pmatrix} \cos t - \sin t \\ 2\cos t \end{pmatrix} + i\,e^t \begin{pmatrix} \cos t + \sin t \\ 2\sin t \end{pmatrix}$$

And a general solution is:

$$\hat{Y} = k_1 e^t \begin{pmatrix} \cos t - \sin t \\ 2\cos t \end{pmatrix} + k_2 e^t \begin{pmatrix} \cos t + \sin t \\ 2\sin t \end{pmatrix}$$

So a particular solution satisfying $(1,1)$ is:

$$\hat{Y} = \frac{1}{2} e^t \begin{pmatrix} \cos t - \sin t \\ 2\cos t \end{pmatrix} + \frac{1}{2} e^t \begin{pmatrix} \cos t + \sin t \\ 2\sin t \end{pmatrix}$$