Examlet 4 Foundations of Advanced Math 4/14/06

Each problem is worth 10 points. Appropriate justification is required for full credit.

1. a) Let R be a relation on the set A. State the definition of R being transitive.

   b) Give an example of a relation on the set $\{a, b, c\}$ which is reflexive and symmetric, but not transitive.

2. Let $A = \{\heartsuit,\diamondsuit,\clubsuit,\spadesuit\}$. Let $R = \{(\heartsuit,\diamondsuit),(\spadesuit,\clubsuit)\}$. Is R reflexive? Symmetric? Transitive?

3. If R and S are symmetric relations on a set A, then $R \cap S$ is a symmetric relation on A.

4. Define a relation ~ on the set of ordered pairs of real numbers by

$$(x_1, y_1) \sim (x_2, y_2) \text{ iff } \sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2}\,.$$

a) Find three points which are related to the point (2,0) under ~.

b) Is ~ an equivalence relation on $\mathbb{R} \times \mathbb{R}$?

5. a) Let $f \subseteq A \times B$ be a bijective function. Define f^{-1} in terms of ordered pairs.

b) Let $f \subseteq \mathbb{R} \times \mathbb{R}$ and $g \subseteq \mathbb{R} \times \mathbb{R}$ be functions. Define $f + g$ in terms of ordered pairs.

Extra Credit [2 points possible]: If $f \subseteq A \times B$ and $g \subseteq C \times D$ be functions, then $f \cap g$ is a function from $A \cap C$ to $B \cap D$.