Homework Foundations 1/16/06

The understanding is that n represents an integer, and x a real number, unless otherwise stated.

- Call an integer m **even** if it is equal to $2n$ for some integer n.
- Call an integer m **odd** if it is equal to $2n + 1$ for some integer n.

1. If n is even, then n^2 is even.

2. If n is odd, then n^2 is odd.

3. If n^2 is even, then n is even.

4. If n^2 is odd, then n is odd.

5. The square of an even number is even.

6. The square of an odd number is odd.

- Call an integer m **threven** if it is equal to $3n$ for some integer n.
- Call an integer m **throdd** if it is equal to $3n + 1$ for some integer n.
- Call an integer m **throddodd** if it is equal to $3n + 2$ for some integer n.

7. The sum of two threven integers is threven.

8. The sum of two throdd integers is throddodd.

9. The sum of a throdd and a throddodd integer is threven.

10. The product of a threven integer with a throdd integer is threven.

11. The square of a threven integer is threven.

12. The square of a throdd integer is throdd.

13. The square of a throddodd integer is throdd.

- Let a be an integer. If an integer m is equal to an for some integer n, then we say m is **divisible by *a***.

14. 14 is divisible by 7.

15. 15 is not divisible by 7.

16. If n is divisible by 2 and m is divisible by 3, then $n + m$ is divisible by 5.

17. If n is divisible by 2 and m is divisible by 3, then $n \cdot m$ is divisible by 6.

18. The product of any two consecutive integers is even.

19. The product of any three consecutive integers is threven.

20. The product of any four consecutive integers is divisible by 24.

- For any $n > 0$, the ***n*th triangular number** is the number $\frac{n(n+1)}{2}$.

21. Prove that the sum of any two consecutive triangular numbers is a perfect square.

22. For any $n > 0$, the difference of the n^{th} and $n + 1^{st}$ triangular numbers is $n + 1$.

- A real number is **rational** if it can be written in the form $\frac{a}{b}$ for integers a and b with $b \neq 0$.
- A real number is **irrational** if it is not rational.

23. The sum of two rational numbers is rational.

24. The sum of two irrational numbers is irrational.

25. The product of two rational numbers is rational.

26. The product of two irrational numbers is irrational.

27. Between any two integers there is another integer.

28. Between any two rational numbers there is another rational number.

29. Between any two irrational numbers there is an irrational number.

30. For any integer n, the number $n^2 + n + 17$ is prime.