Homework 2   Foundations   2/13/06

1. Suppose that $a, b \in \mathbb{R}$. If $a < b$, then $a < \frac{a+b}{2} < b$.

2. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b}$.

3. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $\sqrt{ab} \le \frac{a+b}{2}$.

4. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $\sqrt{a^2 + b^2} \le a + b$.

5. Suppose that $a, b \in \mathbb{R}$. Then $|a - b| \ge |a| - |b|$.

6. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $a + c < b + d$.

7. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $ac < bd$.

8. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$ and $b, c > 0$. Then $ac < bd$.

9. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $a, b > 0$. Then $\forall n \in \mathbb{N}$, $a^n < b^n$.

10. Suppose that $a, b \in \mathbb{R}$. If $a^2 = b^2$, then $a = b$.

11. Suppose that r is a real number. Then $r^2 \ge r$ and $\frac{1}{r^2} \le \frac{1}{r}$.

12. Suppose that r is a real number and $r \ge 1$. Then $r^2 \ge r$ and $\frac{1}{r^2} \le \frac{1}{r}$.