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Suppose that N is a set. We call N a **Peano system** iff the following conditions are satisfied:

I. $0 \in N$.

II. For each $x \in N$, there is a unique element $x' \in N$ (we call x' the *successor* of x).

III. $\forall x \in N, x' \neq 0$.

IV. $\forall x, y \in N, x' = y' \Rightarrow x = y$

V. If $M \subseteq N$ for which $0 \in M$ and $\forall x \in M, x' \in M$, then $M = N$.

Given a Peano system N, we make the following definition:

- Given $x, y \in N$, define their **sum** $x + y$ by
  - $x + 0 = x$
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Prove the following statements, given that N is a Peano system.

1. $\forall x, y \in N, x + (y + 0) = (x + y) + 0$.

2. $\forall x, y, z \in N, x + (y + z) = (x + y) + z \Rightarrow x + (y + z') = (x + y) + z'$.

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13. $\forall y \in N$, with $y \neq 0, 0 \neq 0 + y$.

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