Exam 3 Calc 2 3/30/2007

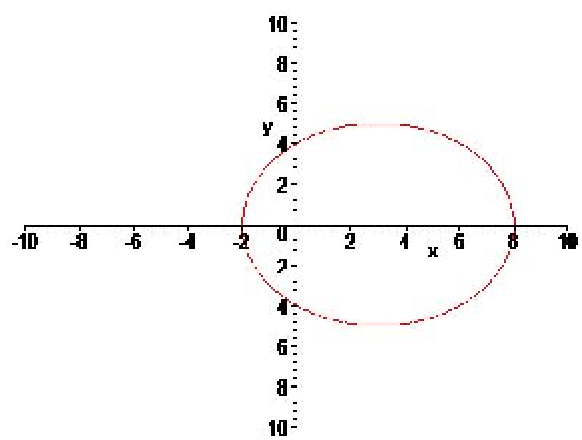
Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Determine whether the sequence $\left\{ \dfrac{2^n}{3^{n+1}} \right\}$ converges or diverges, and if it converges find its limit.

2. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$ converges or diverges, and if it converges find its limit.

3. Determine whether $y = x^2 - x^{-1}$ is a solution to the differential equation $xy' + y = 3x^2$.

4. Find an equation for the conic section shown



5. Find the exact coordinates of the highest point on the graph with parametric equations

$$x = \cos t - 2\sin t,\ y = \sin t + \cos t.$$

6. The amount of a particular pollutant (in grams) in a lake which is gradually being clogged with garbage is modeled by the differential equation $\frac{dp}{dt} = 5 - \frac{10p}{200 - 5t}$, where t is measured in years. Use Euler's method with steps of size $\Delta t = 10$ to approximate to the nearest tenth of a gram the amount of pollutant present after 20 years if $p(0) = 0$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man! This math in college is so [bleep] hard! Our AP teacher in high school told us if you get a negative for area, you just take the negative away 'cause area's always positive, but I did that on our quiz about the parameter things and it got marked wrong. The professor even said after that in class that you couldn't just take off the negative, but I couldn't really get what he was saying you could do. I guess I'm screwed."

   Help Biff by explaining to him what alternatives might be better than just removing negative signs or giving up.

8. Find the solution of the differential equation $\frac{dy}{dx} = y^2 + 1$ that satisfies the initial condition $y(0) = 2$.

9. Find the exact area inside both $r = 2\cos 3\theta$ and $r = 2\sin 3\theta$.

10. Set up an integral or integrals for the area of the first-quadrant portion of the curve with parametric equations $x = 3\cos t - \sin t$, $y = 3\cos t + \sin t$.

Extra Credit (5 points possible): Suppose that you begin with the unit interval [0,1] on the number line. In step one, remove the (open) middle third, (1/3, 2/3), leaving [0,1/3] and [2/3,1]. In the second step, remove the (open) middle thirds of each of these intervals, leaving [0,1/9], [2/9,1/3], [2/3,7/9], and [8/9,1]. In the third step remove the (open) middle thirds of each remaining interval, etc. What is the total length removed after infinitely many steps have been completed?