Exam 4 Calc 2 4/20/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. What is the 4^{th} degree MacLaurin polynomial for e^x?

2. Give an example of a series which converges, but does not converge absolutely.

3. Show that $\sum_{n=1}^{\infty} \frac{1}{n^3+2}$ converges.

4. Determine whether $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n}{\ln n}$ converges or diverges.

5. Determine whether $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges.

6. Find the radius of convergence of the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is *so* unfair. Our professor keeps saying there's more than one way to do one of these problems, right? Like, you know there's lots of different tests, but maybe there's two of them that both work on the same problem, right? So then on our quiz I got counted wrong for doing a different way from the professor. He said you had to use the Alternating Test, but instead I did it where you take off the negatives, right, like the absolute value of the series? So I showed that the absolute value of the series diverges, so the series has to diverge too, right? But he says I had to do it the Alternating Test way like he did, which is totally hypocritical if he says there's more than one way. I think I'm gonna see if Daddy will sue him."

   Help Bunny by explaining to her either what problems might exist with her approach, or how to defend it to her professor (short of litigation).

8. Find the 3rd degree Taylor Polynomial (centered at $x = 0$) for $f(x) = \tan x$.

9. Use a 5^{th} degree power series to give an approximation of ln 1.2. State your answer to five decimal places.

10. Use a Taylor polynomial of degree 9 for $\sin(x^3)$ to approximate $\int_0^{0.5} \sin(x^3)\,dx$ to eight decimal places.

Extra Credit (5 points possible):

What is $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{36}+\frac{1}{64}+\frac{1}{128}+\frac{1}{216}+\frac{1}{256}+\ldots$?