Examlet 2b Foundations of Advanced Math 2/23/07

1. a) If A and B are sets, state the definition of $A \cup B$.

b) Let $C = \{1,2,3\}$ and $D = \{3,4,5\}$. What is $C \cap D$?

c) Let $E = [1,5]$ and $F = [3,8]$. What is $E - F$?

2. a) Suppose $A_i = [1/n, n + 3]$ for all $n \in \mathbb{N}$. What is $\bigcup_{n \in \mathbb{N}} A_n$?

b) Let $A_i = [1/n, n + 3]$ for all $n \in \mathbb{N}$ as in part a. What is $\bigcap_{n \in \mathbb{N}} A_n$?

c) Let $B = \{a, b, c\}$ and $C = \{1, 2\}$. What is $B \times C$?

3. a) Prove or give a counterexample: If $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$, then $a + c < b + d$.

b) Prove or give a counterexample: If $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$, then $a - c < b - d$.

4. Let $\{A_i \mid i \in I\}$ be an indexed family of sets, and let B be any set, all subsets of some universal set. Show that $B \cup \bigcup_{i \in I} A_i = \bigcup_{i \in I} (B \cup A_i)$.

5. Let A, B, C, and D be sets. Show that if $A \subseteq B \cap C$, then $A - D \subseteq B$.