Examlet 2c   Foundations of Advanced Math   2/23/07

1. a) If A and B are sets, state the definition of $A \cap B$.

b) Let $C = \{1,2,3\}$ and $D = \{3,4,5\}$. What is $C \cup D$?

c) Let $E = [1,5]$ and $F = [3,8)$. What is $E - F$?

2. a) Suppose $A_i = [1/n, n + 3]$ for all $n \in \mathbb{N}$. What is $\bigcup_{n\in\mathbb{N}} A_n$?

b) Let $A_i = [1/n, n + 3]$ for all $n \in \mathbb{N}$ as in part a. What is $\bigcap_{n\in\mathbb{N}} A_n$?

c) Let $B = \{a, b, c\}$ and $C = \{1, 2\}$. What is $B \times C$?

3. a) Prove or give a counterexample: If $a, b \in \mathbb{R}$, with $a < b$, then $a < \frac{a+b}{2} < b$.

b) Prove or give a counterexample: If $a, b, c, d \in \mathbb{R}$, with $a < b$ then $\sqrt{ab} \leq \frac{a+b}{2}$.

4. Let $\{A_i \mid i \in I\}$ be an indexed family of sets, all subsets of some universal set. Show that

$$\left(\bigcup_{i\in I} A_i\right)' = \bigcap_{i\in I} A_i' .$$

5. Let A, B, and C be sets. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.