Examlet 4b   Foundations of Advanced Math   4/13/07

1. a)   Let $A = \{5, 6, 7, 8, 9\}$ and $R_a = \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7),$
$(8,8),(9,9)\}$. What partition P_a corresponds to the relation R_a on P_a?

b) Let A be as above, and let $P_b = \{\{5,8\}, \{6,7,9\}\}$. What relation corresponds to the partition P_b?

2. a) Give an example of a relation on the set $\{1,2,3,4,5\}$ that is reflexive, symmetric, and transitive.

b) Give an example of a relation on the set $\{1,2,3,4,5\}$ that is reflexive and symmetric but not transitive.

3. Consider the relation $\blacktriangleright$ on $\mathbb{R}$ defined by $x \blacktriangleright y \Leftrightarrow x \geq y + 1$. Determine whether $\blacktriangleright$ is reflexive, symmetric, or transitive, and justify your conclusions clearly.

4. Let $\mathcal{F}$ be a partition of a set A. Define a relation R on A by

$$(a,b) \in R \Leftrightarrow (\exists\, X \in \mathcal{F})\, [a, b \in X]$$

Show that R is an equivalence relation on A.

5. a) Regarding the functions $f : A \to B$ and $g : B \to C$ as subsets of $A \times B$ and $B \times C$, respectively, write a definition of the composition function $g \circ f$.

b) Let A and B be sets. Write the definition of a constant function $h{:}A \to B$ as a set of ordered pairs.