Homework 5 Foundations 4/13/07

- Let $\mathbb{Z}_n = \{m \in \mathbb{Z} \mid 1 \le m \le n\}$.
- Let A be a set with n elements. A bijection $f : \mathbb{Z}_n \to A$ is called a **permutation**.
- Let A be a set with n elements. A **permutation of size k of n elements** is an injection from $\mathbb{Z}_n$ to B. We denote the number of these distinct injections $\boldsymbol{P(n, k)}$.
- The number of distinct k-element subsets of a set with n elements is denoted $\boldsymbol{C(n, k)}$ or $\binom{n}{k}$, read "n choose k".

1. The number of distinct permutations of a set A having n elements is $n!$

2. Find the number of orders in which 6 books can be arranged on a shelf.

3. Find the number of orders in which a deck of 52 cards can be arranged.

4. $P(n, k) = \dfrac{n!}{(n-k)!}$.

5. Find the number of orders in which 3 runners can finish first, second, and third out of 10.

6. Find the number of orders in which 3 letters from the set {a, e, i, o, u} can be chosen.

7. $P(n, n) = n!$

8. For $n \ge 1$, $P(n, n) = P(n, n-1)$.

9. For $n \ge 1$ and $1 \le k \le n$, $C(n, k) = \dfrac{n!}{k!(n-k)!}$.

10. Find the number of 5-card poker hands that can be dealt from a deck of 52.

11. Find the number of ways 2 faculty members can be chosen from a department of 5 to represent the department at a reception for prospective students.

12. For $n \ge 0$, $C(n, 0) = 1 = C(n, n) = 1$.

13. For $n \ge 1$, $C(n, 1) = n$.

14. For $n \ge 1$ and $1 \le k \le n$, $C(n + 1, k) = C(n, k) + C(n, k - 1)$.

15. For $n \ge 1$ and $1 \le k \le n$, $k \cdot C(n, k) = n\, C(n - 1, k - 1)$.

16. For $n \ge 1$ and $1 \le k \le n$, $(n - k) \cdot C(n, k) = n \cdot C(n - 1, k)$

17. For $n \ge 1$ and $0 \le k \le n$, $C(n, k) = C(n, n - k)$.

- Define **probability** $= \dfrac{\text{number of favorable events}}{\text{number of total events}}$.

18. When a fair coin is tossed, what is the probability of the coin coming up tails?

19. When a fair coin is tossed twice, what is the probability of two heads?

20. When a fair coin is tossed twice, what is the probability of one heads and one tails?

21. When a fair coin is tossed three times, what is the probability of exactly two tails?

22. If a jar contains three balls, one red, one blue, and one white, and two balls are drawn at random from the jar (without replacement), what is the probability that neither ball is blue?

23. If a jar contains five balls, one red, two blue, and two white, and two balls are drawn at random from the jar (without replacement), what is the probability that both balls are white?

24. If a jar contains five balls, one red, two blue, and two white, and two balls are drawn at random from the jar (replacing each after it's drawn), what is the probability that both balls are white?

25. If a jar contains five balls, one red, two blue, and two white, and three balls are drawn at random from the jar (replacing each after it's drawn), what is the probability that the balls are drawn red, blue, and white in that order?

26. If a jar contains five balls, one red, two blue, and two white, and three balls are drawn at random from the jar (replacing each after it's drawn), what is the probability that the balls are drawn red, blue, and white, not necessarily in that order?

27. If two cards are drawn from a standard deck, what is the probability of both cards being clubs?

28. If two cards are drawn from a standard deck, what is the probability of both cards being face cards?

29. If two cards are drawn from a standard deck, what is the probability of both cards being red aces?

30. If a standard die is tossed, what is the probability that the result is at least 5?

31. If a standard die is tossed twice, what is the probability that the total is exactly 5?

32. If a standard die is tossed three times consecutively, what is the probability that each result is higher than the one before?

33. If three standard dice are tossed, what is the probability that the total on all three dice is at least 17?

34. Events are called **independent** when the second event's probability is not affected by the outcome of the first event. Which of the questions about involves independent events?