

Exam 2 Calc 2 2/29/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Show that $\int x \cos x dx = x \sin x + \cos x + C$.

2. State the formula for the surface area obtained by rotating the curve $y = f(x)$, $a \leq x \leq b$, about the x -axis [assuming that $y = f(x)$ is positive for all values of x].

3. Evaluate $\int \sin^3 \theta \cos^2 \theta d\theta$.

4. Set up and evaluate an integral for the arc length of the function $f(x) = x^2/2$ between $(0,0)$ and $(2,4)$. [Hint: You can use the results of problems 8 and 9]

5. Show that if a region shaped like a right triangle with legs of length a and b is positioned so that the right angle is at the origin, the leg of length a lies along the positive x -axis, and the leg of length b lies along the y -axis, then \bar{x} , the x coordinate of the center of mass, lies at $a/3$.

6. The function $f(t) = \begin{cases} 0 & \text{if } t < 0 \\ te^{-t} & \text{if } t \geq 0 \end{cases}$ is a probability density function. Compute the mean for this p.d.f.

7. Biff is a calculus student at Enormous State University, and he has a question. Biff says “Dude, I’m cramming for my calc test, and I think these partial fraction things are really whacked. The test from last year I paid my frat brother \$50 for has this question with, like, $\frac{x^3}{x(x+1)(x-1)}$, and I did the stuff and got $\frac{1}{2}$ and $-\frac{1}{2}$ and 0, so it’s $\frac{0}{x} + \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$, right? But then I looked at the back of the book, and they must have used some of that crazy log property stuff, ‘cause they got the integral to be $x + \frac{1}{2} \ln \left(\frac{|x-1|}{|x+1|} \right) + C$. How’d they do that?”

Help Biff by pointing out any issues with his approach, or suggesting how to make his result match the book’s.

8. Show that $\int \sqrt{a^2 + u^2} du$ can be transformed by an appropriate substitution into $\int a^2 \sec^3 \theta d\theta$.

9. Derive line 77 from our table of integrals.

10. Evaluate $\int \left(\frac{13}{(x^2 + 4)(x - 3)} \right) dx$.

Extra Credit (5 points possible): Derive line 120 on our table of integrals.