Exam 3 Calc 2 3/28/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Find the first 4 partial sums of the series $\sum_{n=1}^{\infty} \frac{3}{10^n}$.

b) Find the sum of the series in part a.

2. a) Convert the rectangular coordinates (–5, 0) to polar coordinates.

b) Convert the polar coordinates (6, $\pi/3$) to rectangular coordinates.

3. Find an equation for the line tangent to the curve with parametric equations $x = t^4 + 1$, $y = t^3 + t$ at the point where $t = 2$.

4. Determine whether the sequence $a_n = \dfrac{3+5n^2}{n+n^2}$ converges or diverges, and if it converges find the limit.

5. A cup of coffee initially has a temperature of 95° C, and is left in a 20° C room . Suppose that you also know that the coffee will cool at a rate of 1° C each minute when its temperature is 70° C.
   a) Write a differential equation for the temperature of the coffee after t minutes.

   b) Use Euler's method with a step size of $\Delta t = 2$ minutes to approximate the temperature of the coffee after 4 minutes.

6. Identify the curve with equation $2y^2 - 3x^2 - 4y + 12x + 8 = 0$, and sketch a good graph of it.

7. Biff is a calculus student at Enormous State University, and he has a question. Biff says "Dude, I love these parametric things, 'cause it's like all you gotta do is have your calculator graph 'em, you know? But for this one I think it screwed up somehow, 'cause it's $x = 3\cos t$ and $y = 3\sin t$, but the graph comes up like kind of a circle. That can't be right, 'cause trig stuff is all wavy, right?"

   Help Biff by explaining what's going on.

8. Find a solution to the differential equation $\frac{dH}{dt} = k\left(H - A\right)$ satisfying the initial condition $H(0) = H_0$.

9. a) Find the area inside both of the curves $r = \cos 2\theta$ and $r = \sin 2\theta$.

b) Let n be a natural number, with $n \geq 2$. Find the area inside both of the curves $r = \cos n\theta$ and $r = \sin n\theta$.

10. Set up integrals for the area inside the loop of the graph of the function given parametrically by $x = t^3 - 9t$, $y = t^3 - 4t^2 + 3t$.

Extra Credit (5 points possible): Define a sequence by letting $a_1 = 3$ and then letting $a_{n+1} = \sqrt{a_n + 1}$. Does a_n converge?