

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. For what values of k does the function $y = \cos kt$ satisfy the differential equation $y'' + 4y = 0$?

$$y' = k(-\sin kt) = -k \sin kt$$

$$y'' = -k^2 \cos kt$$

$$y'' + 4y = 0$$

$$(-) \quad -k^2 \cos kt + 4 \cos kt = 0$$

$$(-) \quad \cos kt (4 - k^2) = 0$$

$$(-) \quad \left[\begin{array}{l} \cos kt = 0 \rightarrow kt = \frac{\pi}{2} \rightarrow k \text{ depends on } t \\ 4 - k^2 = 0 \rightarrow k = \pm 2 \end{array} \right.$$

$$\rightarrow \boxed{k = \pm 2}$$

Excellent!

2. Is $y = t e^{-t}$ a solution to $y'' + 2y' + y = 0$?

$$y' = 1 \cdot e^{-t} + t \cdot (-1) \cdot e^{-t} = e^{-t} - t \cdot e^{-t}$$

$$y'' = -e^{-t} - (1 \cdot e^{-t} + t \cdot (-e^{-t}))$$

$$= -e^{-t} - e^{-t} + t \cdot e^{-t}$$

$$= -2e^{-t} + t \cdot e^{-t}$$

$$y'' + 2y' + y = 0$$

$$(-) \quad -2e^{-t} + t \cdot e^{-t} + 2e^{-t} - 2t \cdot e^{-t} + t \cdot e^{-t} = 0$$

$$(-) \quad 0 = 0$$

So yes! $y = t \cdot e^{-t}$ is a solution

Great!