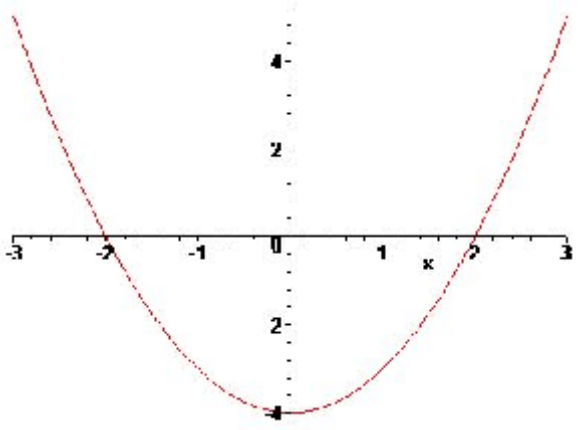
Exam 1 Differential Equations 2/8/08

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Determine whether the function $y = \sin t$ is a solution to the differential equation $\frac{d^2y}{dt^2} + y = \sin t$.

2. State the definition of a separable differential equation.



3. Sketch the phase line for the differential equation $dy/dt = f(y)$ if $f(y)$ has the graph shown:

4. Find a general solution to the differential equation $\frac{dy}{dt} = t + t\,y^2$.

5. Use Euler's method with step size $\Delta t = 0.5$ to approximate $y(1)$ to the nearest hundredth for a solution y to the differential equation $\frac{dy}{dt} = 2y + 1$ subject to the initial condition $y(0) = 3$.

6. Find a general solution to the differential equation $\frac{dy}{dt} = \frac{y}{t} + 4$.

7. Suppose that $\frac{dy}{dt} = f(y)$ is a differential equation satisfying the hypotheses of our existence and uniqueness theorems. Further suppose that $y_1(t) = 0$, $y_2(t) = 20$, and $y_3(t) = 30$ are all solutions for all t. If you're seeking a solution satisfying the initial condition $y(0) = 5$, what can you conclude about that solution?

8. Find the power series expansion for the general solution up to degree four to the differential equation $\frac{d^2y}{dt^2} + y = \sin t$.

9. **Sketch the bifurcation diagram** for the differential equation $\frac{dy}{dt} = y^3 + \alpha y^2$. Include direction arrows on the phase lines and make clear the exact α values where bifurcations occur.

10. For what value(s) of the parameter r is it possible to find explicit formulas (without integrals) for the solutions to $\frac{dy}{dt} = t^r y + 4$?