Exam 2a   Differential Equations   3/7/08

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Determine whether $x(t) = 2e^{4t} - 6e^{t}$, $y(t) = 2e^{4t} + 3e^{t}$ is a solution to the system

$$\begin{aligned} \frac{dx}{dt} &= 2x + 2y \\ \frac{dy}{dt} &= 1x + 3y \end{aligned}.$$

2. State (you don't need proof) the Laplace transforms:

a) $\mathcal{L}\left[e^{at}\right]$

b) $\mathcal{L}\left[u_a\right]$

3. Consider the system

$$\frac{dx}{dt} = 10(y - x)$$
$$\frac{dy}{dt} = 28x - y - xz\,.$$
$$\frac{dz}{dt} = -\tfrac{8}{3}z + xy$$

Find all equilibria of the system.

4. Write a system of differential equations to model two interacting populations (of creatures called *pinks* and *quinks*) represented by $p(t)$ and $q(t)$ with the following characteristics:

- pinks, in the absence of quinks, will experience exponential population decay.
- quinks, in the absence of pinks, have their population grow logistically to a carrying capacity of Q.
- interactions between pinks and quinks benefit the pink population.
- interactions between pinks and quinks hurt the quink population.

Use letters of your choice for proportionality constants, but write your system in such a way that all of these parameters have positive values.

5. Suppose that you know $x(t) = k_2 e^{2t} - \frac{k_1}{3} e^{-t}$, $y(t) = k_1 e^{-t}$ is a general solution to a system of differential equations. Find the solution satisfying the initial condition $\mathbf{Y}(0) = (x(0), y(0)) = (-1, 3)$.

6. Find a solution to the system $\begin{aligned} \frac{dx}{dt} &= x + 2y \\ \frac{dy}{dt} &= 3x \end{aligned}$.

7. Show that $\mathcal{L}\left[t\right]=\frac{1}{s^2}$, and note any necessary restrictions.

8. Compute the inverse Laplace transform $\mathcal{L}^{-1}\left[\frac{4}{s(s+3)}\right]$.

9. Show that $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2 \cdot L\ [y] - s \cdot y(0) - y'(0)$.

10. Suppose that you know that $\mathbf{Y}_1(t) = (x(t), y(t))$ is a solution to the system of equations

$$\frac{dx}{dt} = 3x - 5y$$
$$\frac{dy}{dt} = 2x + y$$

. Show that $\mathbf{Y}_2(t) = (a \cdot x(t), a \cdot y(t))$ is also a solution, for any real number a.