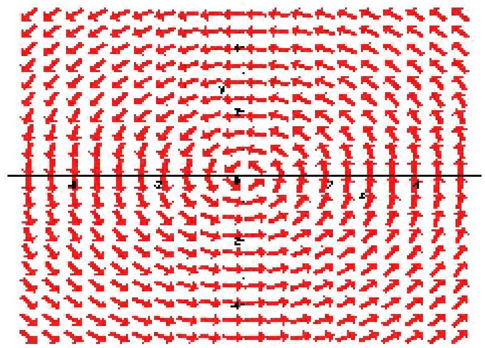
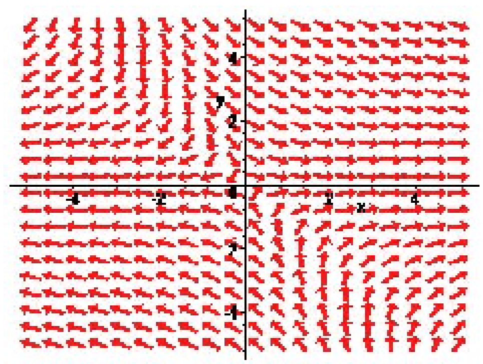
Exam 3 Differential Equations 4/18/08

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. One of the planar systems whose phase plane is shown at right has two distinct real nonzero eigenvalues, one has two purely imaginary eigenvalues, and one has two real eigenvalues, one of which is zero. Identify which is which.





2. Give an example of a planar linear system where the origin is a saddle.

3. If a planar system of differential equations has eigenvalues $\lambda_1 = 3$, $\lambda_2 = 0$ and associated eigenvectors $\mathbf{v}_1 = (1,0)$ and $\mathbf{v}_2 = (3,-1)$, write a general solution to the system.

4. The system

$$\frac{dx}{dt} = -2x - 1y$$
$$\frac{dy}{dt} = 1x - 4y$$

has the single eigenvalue $\lambda = -3$, with corresponding eigenvector (1,1). Find a solution to this system satisfying the initial condition $\mathbf{Y}_0 = (2,2)$.

5. Find a general solution to the equation $y'' + 3y' - 4y = 0$.

6. Find a general solution to the equation $y'' + 3y' - 4y = e^{5t}$.

7. Given that the system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}\mathbf{Y}$ has eigenvalues $\lambda = 4 \pm 2i$, and an eigenvector corresponding to $\lambda = 4 + 2i$ is $\begin{pmatrix} 1 \\ 1+i \end{pmatrix}$, write a general solution to the system.

8. Find a general solution to the equation $y'' + 9y = 2\sin 3t$.

9. Suppose that $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$ has a coefficient matrix of the form $A = \begin{pmatrix} 0 & 1 \\ a & 2b \end{pmatrix}$. For what values of a and b will the origin be a source?

10. Show that if λ is an eigenvalue of a matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{v}$, then $\mathbf{Y} = e^{\lambda t}\mathbf{v}$ is a solution to $\dfrac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$.