Examlet 4 Foundations of Advanced Math 4/11/08

1. a) State the definition of a transitive relation.

   b) Give an example of a relation on the set $\{1, 2, 3\}$ which is reflexive but not symmetric.

2. a) Suppose that $\equiv$ is the relation on the set $A = \{a, b, c, d, e\}$ defined by $\equiv\ = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (e,e)\}$. Write the equivalence classes corresponding to $\equiv$ out explicitly.

   b) Suppose that P is the partition $\{\{a\}, \{b, d\}, \{c, e\}\}$ of the set $A = \{a, b, c, d, e\}$. Find the relation R corresponding to P.

3. Let R be a relation on $\mathbb{Z}$ defined by $x \ R \ y \Leftrightarrow y \neq 5$. Determine whether R is reflexive, symmetric, or transitive, and support your conclusions well.

4. Let m be a natural number, and let $\equiv_m = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} \mid a - b = km \text{ for some } k \in \mathbb{Z}\}$. Show that $\equiv_m$ is an equivalence relation (this relation is usually called *congruence modulo m*).

5. a) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of f being onto.

b) Recall χ_B, the characteristic function of a set B, from chapter 4; it was defined then by

$$\chi_B(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{if } x \notin B \end{cases}$$

Let A be a set and B be a subset of A. Write χ_B as a subset of $A \times \{0,1\}$.