**Problem Set 3 Foundations Due 2/8/2008**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Consider the sum $a^1 + a^2 + a^3 + a^4 + ... + a^n$.
   a) Write the sum in sigma notation.
   b) Find a formula for the sum.
   c) Prove that your formula works for all $n \in \mathbb{N}$.

2. Suppose that you have a collection of sets $S_1, S_2, S_3, S_4, ..., S_n$, such that for any $a, b \in \mathbb{N}$ with $a, b \leq n$, there exists some $x$ for which $x \in S_a$ and $x \in S_b$. What can you conclude about $S_1 \cap S_2 \cap S_3 \cap S_4 \cap ... \cap S_n$? Defend your claim.

3. Consider the formula $1+2+3+...+n=\dfrac{n^2+n+1}{2}$.
   a) Write the formula in sigma notation.
   b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.
   c) Explain why mathematical induction does **not** prove that this formula is true for all $n \in \mathbb{N}$.

4. Do problem #10 in §3.1.

5. Do problem #11 in §3.1.

6. Prove part 5 of Theorem 3.2.1 by a set-theoretic proof.

7. Prove part 7 of Theorem 3.2.1 by a set-theoretic proof.

8. Let $A \subseteq B$. Show that $\{c\} \cup A \subseteq \{c\} \cup B$.