Problem Set 8 Foundations Due 4/4/2008

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Let R be the relation on $\mathbb{Z}$ defined by $x\ R\ y$ iff $x - y$ is throdd.
   a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.
   c) Determine whether R is an equivalence relation on $\mathbb{Z}$. Support your answer well.

2. Let ~ be the relation on $\mathbb{R}$ defined by $x \sim y$ iff $x - y \in \mathbb{Z}$.
   a) Pick an element t of $\mathbb{R}$ and find three other elements of $\mathbb{R}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{R}$ which are not related to it.
   c) Determine whether ~ is an equivalence relation on $\mathbb{R}$. Support your answer well.

3. Let $\approx$ be the relation on $\mathbb{R}$ defined by $x \approx y$ iff $x - y \in \mathbb{Q}$.
   a) Pick an element t of $\mathbb{R}$ and find three other elements of $\mathbb{R}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{R}$ which are not related to it.
   c) Determine whether $\approx$ is an equivalence relation on $\mathbb{R}$. Support your answer well.

4. Let $\simeq$ be the relation on $\mathbb{Z}$ defined by $x \simeq y$ iff $x - y = 5$.
   a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.
   c) Determine whether $\simeq$ is an equivalence relation on $\mathbb{Z}$. Support your answer well.

5. Let R be the relation on the set P($\mathbb{R}$) of all polynomials with real coefficients defined by $f \sim g$ iff f and g have a root in common.
   a) Pick an element t of P($\mathbb{R}$) and find three other elements of P($\mathbb{R}$) which are related to it.
   b) For your element t from part a, find three other elements of P($\mathbb{R}$) which are not related to it.
   c) Determine whether ~ is an equivalence relation on P($\mathbb{R}$). Support your answer well.

6. Let $\bowtie$ be the relation on the set $R(I)$ of integrable functions from [0,1] to $\mathbb{R}$ defined by $f \bowtie g$ iff
   $$\int_0^1 f(x)\,dx = \int_0^1 g(x)\,dx.$$
   a) Pick an element h of $R(I)$ and find three other elements of $R(I)$ which are related to it.
   b) For your element h from part a, find three other elements of $R(I)$ which are not related to it.
   c) Determine whether $\bowtie$ is an equivalence relation on $R(I)$. Support your answer well.

7. Suppose that $\equiv$ is the relation on the set $A = \{a, b, c, d, e\}$ defined by $\equiv\ = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (e,e)\}$. What are the equivalence classes corresponding to $\equiv$?

8. Suppose that P is the partition $\{\{a\}, \{b, d\}, \{c, e\}\}$ of the set $A = \{a, b, c, d, e\}$. Find the relation R corresponding to P.