Problem Set 9 Foundations Due 4/18/2008

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. For $n \geq 1$ and $1 \leq k \leq n$, $C(n, k) = \dfrac{n!}{k!(n-k)!}$.

2. The Banana Theorem: Let A be a set with n elements of k different types (such that elements of the same type are regarded as indistinguishable from one another for purposes of orderings). Let n_i be the number of elements of type i for each integer i from 1 to k. Then the number of different arrangements of the elements in A will be $\dfrac{n!}{\prod_{i=1}^{k}(n_i!)}$.

3. What is the probability of getting exactly two heads among three tosses of a coin?

4. What is the probability of drawing (without replacement) two hearts from a standard deck of 52 cards?

5. What is the probability of having at least two identical letters in a randomly selected string of four letters?

6. What is the probability that when a die is tossed twice, the result will be the same both times?

7. How many numbers between 1 and 1,000,000 have no 9's among their digits?

8. How many distinguishable fruit baskets with 7 items can be created using apples, oranges, and pears?