**Exam 2 Calc 2 3/6/2009**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Find $L_2$ for $\int_0^{0.5} e^{-x^2}\,dx$.

   b) Find $M_2$ for $\int_0^{0.5} e^{-x^2}\,dx$.

2. Set up an integral for the area of the surface obtained by rotating the portion of $y = x^3$ on the interval [0,2] about the $x$- axis.

3. Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 - x + 1}{x(x-2)(x^2+x+1)(x^2+1)^3}.$$

4. Evaluate $\int_1^{\infty} \frac{1}{x^3}\,dx$ .

5. Show that $\int u^4 \sqrt{a^2 - u^2}\, du$ can be transformed by an appropriate substitution into $a^6 \int \sin^4 q \cos^2 q \, dq$ .

6. Find the length of the curve $y$ = ln(sec $x$) on the interval [0, p/4].

7. Derive line 84 on the table of integrals.

8. You have been tasked with writing a section for the forthcoming book *Incredibly Rarely Used Techniques in Calculus*. The section is to cover integrating combinations of $\csc x$ and $\cot x$. Explain, in terms a typical calculus student can follow, a basic procedure for integrating products of powers of these functions.

9. Suppose that $p(t) = \begin{cases} \frac{1}{4}e^{-t/4} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$ is a p.d.f. representing a probability that a computer armoire purchased from Home Design Solutions breaks within $t$ weeks of purchase.

a) Find the median of this p.d.f.

b) A cumulative distribution function $c(t)$ associated with a given p.d.f. $p(t)$ is a function which, for each value of $t$, gives the proportion of the sample less than $t$. Find $c(t)$ for Home Design Solutions computer armoires.

10. Consider the trapezoidal region bounded by $x = 0$, $y = 0$, $x = 1$, and a line with $y$-intercept 1 and slope $m$.
   a) If $m = 1$, find the $x$ coordinate of the center of mass of the trapezoidal region.
   b) For other positive constant values of $m$, how large can the $x$ coordinate of the center of mass of the region get?

Extra Credit (5 points possible):
Derive line 117 on the table of integrals.