Examlet 1 Foundations of Advanced Math 2/6/09

1. a) Determine whether the propositionals $P \Rightarrow Q$ and $\neg Q \Rightarrow \neg P$) are equivalent.

   b) Determine whether the propositionals $(P \wedge Q) \Rightarrow R$ and $(P \Rightarrow R) \vee (Q \Rightarrow R)$ are equivalent.

2. If n divides a and n divides b, then n divides $a + b$.

3. $\sqrt[3]{2}$ is irrational.

4. Prove that $\forall n \in \mathbb{N}, \sum_{r=1}^{n}(2r-1)=n^2$.

5. We say that an integer m is **congruent to 0 modulo 5** iff $m = 5n$ for some integer n.
We say that an integer m is **congruent to 1 modulo 5** iff $m = 5n + 1$ for some integer n.
We say that an integer m is **congruent to 2 modulo 5** iff $m = 5n + 2$ for some integer n.
We say that an integer m is **congruent to 3 modulo 5** iff $m = 5n + 3$ for some integer n.
We say that an integer m is **congruent to 4 modulo 5** iff $m = 5n + 4$ for some integer n.

a) If a is congruent to 1 modulo 5, then a^2 is congruent to 1 modulo 5.

b) If a is an integer for which a^2 is congruent to 1 modulo 5, then a is congruent to 1 modulo 5.