inters. in both

1. a) Find $\{0,1,3,4\} \cap \{0,2,4\}$

$\{0,4\}$

union

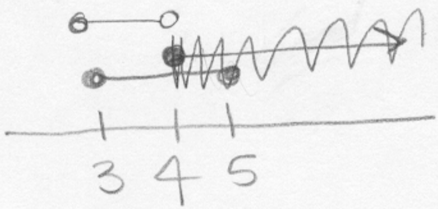
b) Find $\{0,1,3,4\} \cup \{0,2,4\}$

$\{0,1,2,3,4\}$

in ≠ not in

c) Find $[3,5] - [4, \infty)$

$[3,4)$



d) State the definition of the Cartesian product of two sets A and B.

$A \times B = \{(x,y) \mid x \in A \wedge y \in B\}$

e) Find $\{1, 2, 3\} \times \{a, b\}$.

$\{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

2. a) For any $a, b \in \mathbb{R}$, $|a - b| \leq |a + b|$.

That's not in general true!

Let $a = 5$ and $b = -2$.

Then $|a-b| = |5 - {}^-2| = |7| = 7$

but $|a+b| = |5 + {}^-2| = |3| = 3$,

yet $7 \nleq 3$.

So we have a counterexample.

b) For any $a \in \mathbb{R}$, $|a| \geq a$.

Consider two cases:

If $a \geq 0$, $|a| = a$, and $a \geq a$.

If $a < 0$, $|a| = -a$. Since $a < 0$, $-a > 0$, so we have $-a \geq 0$ and $0 \geq a$, so $|a| = -a \geq 0 \geq a$, or by the transitive property $|a| \geq a$.

So in either case we have $|a| \geq a$, as desired.

3. Let $A \subseteq B$. Show that $A \cup C \subseteq B \cup C$.

Proof: let x be an element of $A \cup C$. x is therefore an element of A or C. if $x \in A$, $x \in B$ since $A \subseteq B$. since $x \in B$, $x \in B \cup C$. If x were instead an element of C, then x is an element of $B \cup C$. Since all elements of $A \cup C$ are elements of $B \cup C$, $A \cup C \subseteq B \cup C$ as was to be shown $\square$

Excellent

4. Let $\{ A_i \mid i \in I \}$ be an indexed family of sets, and let B be a set. Show that .

$$\left(\bigcup_{i \in I} A_i \right)' = \bigcap_{i \in I} A_i'$$

$$
\begin{aligned}
x \in \left(\bigcup_{i \in I} A_i \right)' \quad & \text{iff} \quad \neg (\exists i \in I)[x \in A_i] \\
& \text{iff} \quad (\forall i \in I)[x \notin A_i] \\
& \text{iff} \quad (\forall i \in I)[x \in A_i'] \\
& \text{iff} \quad \bigcap_{i \in I} A_i
\end{aligned}
$$

Nice!

Since the biconditional is true,

$$\left(\bigcup_{i \in I} A_i \right)' = \bigcap_{i \in I} A'_i. \quad \square$$

5. a) Suppose that $r \in \mathbb{R}$, with $r > 1$. Show that $r^2 > 1$.

we know $r > 1 \quad \ldots (i)$

$\Rightarrow \quad r \cdot r > 1 \cdot r \quad *$

$*$ Since $r > 0$

$\{ 1 > 0 \; ; \; r > 1$

thus $r > 1 > 0 \}$

$\Rightarrow \quad r^2 > r \quad \ldots (ii)$

combining (i) & (ii)

$r^2 > r > 1$

Good.

$\Rightarrow \quad r^2 > 1$

(proved)

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b) Suppose that $r \in \mathbb{R}$, with $r > 1$. Show that for all $n \in \mathbb{N}$, $r^n > 1$.

Proof by Induction

Case 1> when $n = 1$

$r^1 > 1 \Rightarrow \boxed{r > 1}$ (Given as true)

Excellent!

let $n = k$

Case 2> $\boxed{r^k > 1}$ Inductive Hypothesis.

for the case $n = k+1$

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$\boxed{r^k > 1}$; since $r > 0$

$r^k \cdot r > 1 \cdot r$

$\Rightarrow \quad r^{k+1} > r$ but, $r > 1$ (given)

thus $r^{k+1} > r > 1$ or, $\boxed{r^{k+1} > 1}$

true for $n = 1$

true for $n = k$

true for $n = (k+1)$

thus by the process of induction

$r^n > 1$ is true for all $n \in \mathbb{N}$

(note: given $r > 1$)