

**Examlet 3      Foundations of Advanced Math      3/27/09**

1. Let  $a$  and  $b$  be real numbers with  $a < b$ .

a) Give an example of a bijection from  $[0,1]$  to  $[a,b]$ .

b) Give an example of a function from  $[0,1]$  to  $[a,b]$  which is not a bijection.

2. Let  $f: A \rightarrow B$  and  $g: B \rightarrow C$  be injective functions. Show that  $g \circ f$  is injective.

3. Let  $f: A \rightarrow B$ . What is  $f \circ f^{-1}$ ? Support your answer.

4. a) Show that the set of even natural numbers is denumerable.

b) Show that if  $A$  and  $B$  are two denumerable sets, then there exists a bijection from  $A$  to  $B$ .

5. Suppose that  $f:A \rightarrow B$  is a bijection. Show that  $f$  is invertible.