Examlet 4b   Foundations of Advanced Math   4/17/09

1. a) State the definition of a symmetric relation.

   b) Give an example of a relation on the set $\{1, 2, 3\}$ which is reflexive but not transitive.

2. a) Suppose that $\equiv$ is the relation on the set $A = \{a, b, c, d, e\}$ defined by $\equiv\ = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (d,e), (e,d), (e,e)\}$. Write the equivalence classes corresponding to $\equiv$ out explicitly.

   b) Suppose that P is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Find the relation R corresponding to P.

3. Let R be a relation on a set A which is reflexive, symmetric, and transitive; let S be some other relation on A.
   a) Will $R \cup S$ be reflexive?

   b) Will $R \cap S$ be symmetric?

   c) Will $R \cup S$ be transitive?

4. Let R be the relation on $\mathbb{Z}$ defined by $n \sim m$ iff n and m have a factor (other than ± 1) in common.
   a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.
   b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.
   c) Determine whether $\sim$ is an equivalence relation on $\mathbb{Z}$. Support your answer well.

5. a) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of f^{-1}.

b) Let A be a set. Express the identity function $f{:}A \to A$ as a subset of $A \times A$.