Examlet 4b **Foundations of Advanced Math** **4/17/09**

1. a) State the definition of a symmetric relation.

A RELATION R ON SET A IS SYMMETRIC IFF

$$(\forall a,b \in A)[aRb \rightarrow bRa]$$

Good

b) Give an example of a relation on the set {1, 2, 3} which is reflexive but not transitive.

$$\{(1,1),(2,2),(3,3),(1,3),(3,1)(2,3)\}$$

NOT TRANSITIVE BECAUSE

$(2,3) \wedge (3,1)$ but $(2,1) \notin R$ Great!

$2R3 \wedge 3R1$ but $\neg 2R1$ ■

2. a) Suppose that ≡ is the relation on the set $A = \{a, b, c, d, e\}$ defined by $\equiv\ = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (d,e), (e,d), (e,e)\}$. Write the equivalence classes corresponding to ≡ out explicitly.

$[a] = \{a, b, c\}$
$[b] = \{a, b, c\}$
$[c] = \{a, b, c\}$
$[d] = \{d, e\}$
$[e] = \{d, e\}$

$[a] = [b] = [c] = \{a, b, c\}$
$[d] = [e] = \{d, e\}$

b) Suppose that P is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Find the relation R corresponding to P.

$R = \{(1,1), (2,2), (2,4), (4,2), (4,4), (3,3), (3,5), (5,3), (5,5)\}$

Great

3. Let R be a relation on a set A which is reflexive, symmetric, and transitive; let S be some other relation on A.

a) Will $R \cup S$ be reflexive?

Yes. R contains the ordered pair (a, a) for all $a \in A$; therefore $R \cup S$ will also have those pairs.

Good.

b) Will $R \cap S$ be symmetric?

Not necessarily. Let

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$S = \{(1, 2)\}$

These sets satisfy all the conditions given, but $R \cap S = \{(1, 2)\}$ which is not symmetric.

Great

c) Will $R \cup S$ be transitive?

Not necessarily. Let

A = same as above

R = same as above

$S = \{(2, 3)\}$

$R \cup S = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3)\}$.

$R \cup S$ has as elements $(1, 2)$ and $(2, 3)$, but not $(1, 3)$. $\therefore$ $R \cup S$ is not always transitive.

Nice!

4. Let R be the relation on $\mathbb{Z}$ defined by $n \sim m$ iff n and m have a factor (other than 1) in common.

a) Pick an element t of $\mathbb{Z}$ and find three other elements of $\mathbb{Z}$ which are related to it.

b) For your element t from part a, find three other elements of $\mathbb{Z}$ which are not related to it.

c) Determine whether $\sim$ is an equivalence relation on $\mathbb{Z}$. Support your answer well.

a) $t = 8 \in \mathbb{Z}$

$\left.\begin{matrix} \underline{16} \\ \underline{24} \\ \underline{12} \end{matrix}\right\}$ all have a factor of 4 with 8

b) $\left.\begin{matrix} \underline{3} \\ \underline{5} \\ \underline{7} \end{matrix}\right\}$ none of these have a common factor with 8 (other than 1)

c) equivalence relation $\rightarrow$ reflexive, symmetric, transitive.

transitive $\rightarrow$ $(\forall a, b, c \in A)[aRb \wedge bRc \rightarrow aRc]$

let $\underline{a = 4}$ $\underline{b = 6}$ $\underline{c = 9}$

$4 \sim 6 \wedge 6 \sim 9 \rightarrow 4 \sim 9$

True, factor of 2

True, factor of 3

false, no common factor other than 1

Not transitive.

Because R is not transitive, it is not an equivalence relation. □

Excellent!

5. a) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of f^{-1}.

$$f^{-1} = \{(b,a) \in B \times A \mid (a,b) \in f\}$$

b) Let A be a set. Express the identity function $f: A \to A$ as a subset of $A \times A$.

$$i_A = \{(a,a) \in A \times A \mid a \in A\}$$

Great