Homework 1 Foundations 1/14/09

Unless otherwise stated, m and n represent integers, and x a real number.

- Call an integer m **even** iff it is equal to $2n$ for some integer n.
- Call an integer m **odd** iff it is equal to $2n + 1$ for some integer n.

1. If n is even, then n^2 is even.

2. If n is odd, then n^2 is odd.

3. If n^2 is even, then n is even.

4. If n^2 is odd, then n is odd.

5. The cube of an even number is even.

6. The cube of an odd number is odd.

7. The product of any two consecutive integers is even.

8. The sum of any two consecutive integers is odd.

9. The sum of any two non-consecutive integers is even.

- Call an integer m **threven** iff it is equal to $3n$ for some integer n.
- Call an integer m **throdd** iff it is equal to $3n + 1$ for some integer n.
- Call an integer m **throddodd** iff it is equal to $3n + 2$ for some integer n.

10. The sum of two threven integers is threven.

11. The sum of two throdd integers is throddodd.

12. The sum of a throdd and a throddodd integer is threven.

13. The product of a threven integer with a throdd integer is threven.

14. The product of any three consecutive integers is threven.

15. The square of a threven integer is threven.

16. The square of a throdd integer is throdd.

17. The square of a throddodd integer is throdd.

18. There is no integer whose square is throddodd.

- Let a be an integer. Iff an integer m is equal to an for some integer n, then we say ***a* divides *m***.

19. 7 divides 14.

20. 7 divides 100.

21. If 2 divides n and 3 divides m, then 5 divides $n + m$.

22. If 2 divides n and 3 divides m, then 6 divides $n \cdot m$.

23. If p divides q and q divides r, then p divides r.

24. If p divides q and p divides r, then p divides $q + r$.

25. If n is the product of any four consecutive integers, then 24 divides n.

- For any $n > 0$, the ***n*th triangular number** is the number $\frac{n(n+1)}{2}$.

26. Prove that the sum of any two consecutive triangular numbers is a perfect square.

27. For any $n > 0$, the difference of the n^{th} and $n + 1^{st}$ triangular numbers is $n + 1$.

28. The sum of the $n - 1^{st}$ triangular number and three times the n^{th} triangular number is the $2n^{th}$ triangular number.

- A real number is **rational** iff it can be written in the form a/b for integers a and b.
- A real number is **irrational** iff it is not rational.

29. The sum of two rational numbers is rational.

30. The sum of two irrational numbers is irrational.

31. The product of two rational numbers is rational.

32. The product of two irrational numbers is irrational.

33. Between any two integers there is another integer.

34. Between any two rational numbers there is another rational number.

35. Between any two irrational numbers there is an irrational number.

36. For any integer n, the number $n^2 + n + 17$ is prime.

37. For any prime number n, $2^n - 1$ is prime.