

**Homework 3      Foundations      2/11/09**

1. Suppose that  $a, b \in \mathbb{R}$ . If  $a < b$ , then  $a < \frac{a+b}{2} < b$ .
2. Suppose that  $a, b \in \mathbb{R}$ . If  $a < b$  then  $a^2 < b^2$ .
3. Suppose that  $a, b \in \mathbb{R}$ . If  $a, b > 0$ , then  $a < b \Leftrightarrow a^2 < b^2$ .
4. Suppose that  $a, b \in \mathbb{R}$ . If  $a, b > 0$ , then  $a < b \Leftrightarrow \sqrt{a} < \sqrt{b}$ .
5. Suppose that  $a, b \in \mathbb{R}$ . If  $a, b > 0$ , then  $\sqrt{ab} \leq \frac{a+b}{2}$ .
6. Suppose that  $a, b \in \mathbb{R}$ . If  $a, b > 0$ , then  $\sqrt{a^2 + b^2} \leq a + b$ .
7. Suppose that  $a, b \in \mathbb{R}$ . Then  $|a - b| \geq |a| - |b|$ .
8. Suppose that  $a, b, c, d \in \mathbb{R}$ , with  $a < b$  and  $c < d$ . Then  $a + c < b + d$ .
9. Suppose that  $a, b, c, d \in \mathbb{R}$ , with  $a < b$  and  $c < d$ . Then  $a - c < b - d$ .
10. Suppose that  $a, b, c, d \in \mathbb{R}$ , with  $a < b$  and  $c < d$ . Then  $ac < bd$ .
11. Suppose that  $a, b, c, d \in \mathbb{R}$ , with  $a < b$  and  $c < d$  and  $b, c > 0$ . Then  $ac < bd$ .
12. Suppose that  $a, b, c, d \in \mathbb{R}$ , with  $a < b$  and  $c < d$ . Then  $\frac{a}{c} < \frac{b}{d}$ .
13. Suppose that  $a, b \in \mathbb{R}$ , with  $a < b$  and  $a, b > 0$ . Then  $\forall n \in \mathbb{N}, a^n < b^n$ .
14. Suppose that  $a, b \in \mathbb{R}$ . If  $a^2 = b^2$ , then  $a = b$ .
15. Suppose that  $r$  is a real number. Then  $r^2 \geq r$  and  $\frac{1}{r^2} \leq \frac{1}{r}$ .
16. Suppose that  $r$  is a real number and  $r \geq 1$ . Then  $r^2 \geq r$  and  $\frac{1}{r^2} \leq \frac{1}{r}$ .