Homework 4½ Foundations 2/27/09

- A set A is **bounded** iff $\exists\, M \in \mathbb{R}$ such that $\forall a \in A$, $|a| \le M$.

1. Let A and B be bounded sets. Then $A \cap B$ is bounded.
2. Let A and B be bounded sets. Then $A \cup B$ is bounded.
3. Let A be a set with n elements, where $n \in \mathbb{N}$. Then A is bounded.
4. Let I be an indexing set, and A_i be a bounded set for each $i \in I$. Then $\bigcap_{i \in I} A_i$ is bounded.
5. Let I be an indexing set, and A_i be a bounded set for each $i \in I$. Then $\bigcup_{i \in I} A_i$ is bounded.

- A function $f: D \to \mathbb{R}$ is **bounded** iff there $\exists\, M \in \mathbb{R}$ such that $\forall x \in D$, $|f(x)| \le M$.

6. Let f be a constant function. Then f is bounded.
7. Let f and g be bounded functions. Then $f + g$ is a bounded function.
8. Let f and g be bounded functions. Then $f - g$ is a bounded function.
9. Let f and g be bounded functions. Then $f \cdot g$ is a bounded function.
10. Let f and g be bounded functions. Then $f \div g$ is a bounded function.
11. Let f and g be bounded functions. Then $f \circ g$ is a bounded function.
12. Let $n \in \mathbb{N}$, and f_i be a bounded function for each $i \in \{x \in \mathbb{N} \mid 1 \le x \le n\}$. Then $\sum_{i=1}^{n} f_i$ is a bounded function.
13. Let f_i be a bounded function for each $i \in \mathbb{N}$. Then $\sum_{i=1}^{\infty} f_i$ is a bounded function.
14. Let $f + g$ be a bounded function. Then f and g are bounded functions.