Problem Set 2   Foundations   Due 2/2/2009

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Determine whether the propositionals $\neg(P \Rightarrow (Q \wedge R))$ and $\neg(P \Rightarrow Q) \vee \neg(P \Rightarrow R)$ are equivalent.

2. Determine whether the propositionals $(P \wedge Q) \Rightarrow R$ and $(P \Rightarrow R) \vee (Q \Rightarrow R)$ are equivalent.

3. $\sqrt[3]{2}$ is irrational.

4. There are no integers x and y for which $x^2 = 3y + 5$.

5. If a, b, and c are integers for which $a^2 + b^2 = c^2$, then at least one of a or b must be even.

6. Critique the following "proof" of the statement: $\exists x \in \mathbb{R}\ (\forall y \in \mathbb{R},\ x \geq y)$

   "Proof": Well, take any real number y. Then let $x = y + 1$, and we know x is a real number too because of the closure of the reals under addition. Now we know $1 > 0$, and we can add y to both sides to get $y + 1 > y$. Then it's also true that $y + 1 \geq y$, so $x \geq y$, as desired. □