Problem Set 5 Foundations Due 3/16/2009

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Suppose that A is a set of real numbers and there exists $x_0 \in A$ for which $|x - x_0| \le r$ holds for all $x \in A$. Is it necessarily true that for all $x, y \in A$, we will have $|x - y| \le r$?

2. Suppose that A is a set of real numbers and there exists $x_0 \in A$ for which $|x - x_0| \le r$ holds for all $x \in A$. Is it necessarily true that for all $x, y \in A$, we will have $|x - y| \le 2r$?

3. Let A and B be bounded sets. Then $A \cup B$ is bounded.

4. Let I be an indexing set, and A_i be a bounded set for each $i \in I$. Then $\bigcap_{i \in I} A_i$ is bounded.

5. Let f and g be bounded functions. Then $f \cdot g$ is a bounded function.

- A set $A \times B$ in $\mathbb{R} \times \mathbb{R}$ is **bounded** iff $\exists\, M \in \mathbb{R}$ such that $\forall\, (x, y) \in A \times B$, $\sqrt{x^2 + y^2} \le M$.

6. Show that $[0,1] \times [0,1]$ is bounded.

7. Show that $\{0,1,5\} \times \mathbb{R}$ is not bounded.

8. If $A, B \subseteq \mathbb{R}$ are both bounded, $A \times B$ is bounded.