Problem Set 7   Foundations   Due 4/6/2009

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Jon wants to define a function $f\colon A \to B$ as invertible iff
$$\forall a \in A,\ b \in B \text{ with } f(a) = b,\ \exists\, g\colon B \to A \text{ for which } g(b) = a.$$
Is that reasonable?

2. Let $\{A_i \mid i \in \mathbb{N}\}$ be an indexed family of sets, and suppose that A_i is countable for every $i \in \mathbb{N}$. Let $Z_n = \{m \in \mathbb{N} \mid m \le n\}$. Show that $\bigcup_{i \in Z_n} A_i$ is countable for all $n \in \mathbb{N}$.

3. Determine whether the relation ~ on $\mathbb{R}$ defined by $x \sim y$ iff $xy \le 0$ is reflexive, symmetric, or transitive. Provide defense of your answers.

4. Determine whether the relation ~ on $\mathbb{R}$ defined by $x \sim y$ iff $xy < 0$ is reflexive, symmetric, or transitive. Provide defense of your answers.

5. Do #3 in §5.3.

6. Do #6 in §5.3.