Examlet 2   Foundations of Advanced Math   2/26/10

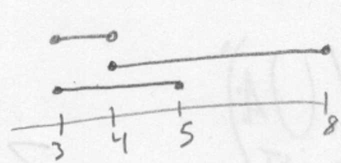
1. a) Let $A = \{1,2,3\}$ and $B = \{2,3,4\}$. What is $A \cup B$?

$A \cup B = \{1,2,3,4\}$

b) Let $A = \{1,2,3\}$ and $B = \{2,3,4\}$. What is $A \cap B$?

$A \cap B = \{2,3\}$

c) Let $C = [3, 5]$ and $D = [4, 8]$. What is $C - D$?



$C - D = [3, 4)$

Good

d) Let $E = \{1,2\}$ and $F = \{5, 7\}$. What is $E \times F$?

$E \times F = \{(1,5),(1,7),(2,5),(2,7)\}$

2. a) Let $\mathbb{N}^+ = \mathbb{N} - \{0\}$. Let $A_n = (0, n)$ for each $n \in \mathbb{N}^+$. What is $\bigcup_{n \in \mathbb{N}^+} A_n$?

All $x \in \mathbb{R}^+$

Yup.

b) Let I be a set such that for each $i \in I$, A_i is itself a set. Then $\left(\bigcup_{i \in I} A_i\right)' = \bigcap_{i \in I} A_i'$.

Let $x \in \left(\bigcup_{i \in I} A_i\right)'$ and so $x \notin \bigcup_{i \in I} A_i \wedge x \in X$.

$\neg \left(x \in A_i \text{ for some } i \in I\right) \wedge x \in X$

$\left(x \notin A_i \text{ for all } i \in I\right) \wedge x \in X$

$\left(x \in A_i' \text{ for all } i \in I\right) \wedge x \in X$

$x \in \bigcap_{i \in I} A_i'$

These steps are reversible so

$\left(\bigcup_{i \in I} A_i\right)' = \bigcap_{i \in I} A_i'$. $\triangle$

Good.

3. a) $\forall x, y \in \mathbb{R}$, If $|x| \le y$, then $-y \le x \le y$.

It is given that $|x| \le y$. This can be multiplied on both sides by -1 to give $-y \le -|x|$. By lemma 1 we know that $-|x| \le x \le |x|$. So we can combine these inequalities to give $-y \le -|x| \le x \le |x| \le y$. By the transitive property this can become $-y \le x \le y$. □

Nice.

b) $\forall x, y \in \mathbb{R}$, If $-y \le x \le y$, then $|x| \le y$.

Let us consider two cases. First, $x \ge 0$, so $|x| = x$. We know from the proposition that $x \le y$. We can subsitute $|x|$ in for x to give $|x| \le y$ which is what we desire. Now consider the second case, $x < 0$ so $|x| = -x$. We know from the proposition that $-y \le x$ so by multipling both sides by -1 we will have $-x \le y$. We substitute in $|x|$ for $-x$ to give $|x| \le y$ which is what we want. So for all cases $|x| \le y$. □

Well done

4. Let A, B, and C be sets. If $A \subseteq B$, then $A - C \subseteq B - C$.

Prop: If $A \subseteq B$, then $A - C \subseteq B - C$.

Proof: Let $x \in A - C$, so that $x \in A$ and $x \notin C$.
We know $A \subseteq B$, so all elements of A are also elements of B; therefore, $x \in B$ as well.
Since $x \in B$, x is an element of $B - C$.
Then all elements of $A - C$ are elements of $B - C$ which is the definition of a subset.

$A - C \subseteq B - C$. $\square$

Great

5. a) $\forall a, b, c, d \in \mathbb{R}$, if $a > b$ and $c > d$, then $a + c > b + d$.

We know $a>b$ and $c>d$.

$a>b$, add c to both sides : $c>d$, add b to both sides

$\underline{a+c>b+c}$: $\underline{c+b>d+b}$

We can put these two inequalities to give us

$a+c>b+c>b+d$

by the transitive property we can then say

$\underline{a+c>b+d}$ $\square$ *Great!*

$-1 \quad -3 \quad 2 \quad -1 \quad -1 \quad 2 \quad -3 \quad 1$

b) $\forall a, b, c, d \in \mathbb{R}$, if $a > b$ and $c > d$, then $a \cdot c > b \cdot d$.

This is not a general rule!

Let $a=1$, $b=-3$, $c=2$, $d=-1$. These values satisfy all the conditions of the prompt.

So, $1 \cdot 2 > (-3)(-1)$

$2 > 3$

Excellent!

We know this to be false so the proposition is false.