

**Exam 1a    Calc 2    2/7/2011**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Let  $F(x) = \int_0^x \sin(t^2) dt$ . What is  $F'(x)$ ?

2. Write an integral representing the average value of the function  $f(x) = \frac{\sin x}{x}$  on the interval  $[\pi/2, 3\pi/2]$ .

3. Consider the region bounded between  $y = 1/x$ , the  $x$ -axis,  $x = 1$ , and  $x = 5$ . Write an integral for the volume of the solid obtained by rotating this region around the  $x$ -axis.

4. Integrate  $\int \frac{x}{\sqrt{1-x^2}} dx$ .

5. Jon's cat Nemo has a plan to fill his guest bathroom with water and stock it with goldfish. The bathroom is a box 4 feet wide, 8 feet long, and 7 feet deep. Write an integral for the amount of work required to pump all of this water up to the top of the room when Jon cleans up the mess (assume a density of  $62.5 \text{ lbs/ft}^3$  for water, and that the goldfish have all mysteriously disappeared before pumping commences).

6. If a spring has a natural length of 20cm, and 12J of work is required to stretch it from 20cm to 40cm, how much work would be required to stretch it from 20cm to 30cm?

7. Biff is a Calc 2 student at Enormous State University. Biff says "Dang, this Calc stuff is killing me. It's getting all, like, theoretical. There was this one question on our exam last week about the average value stuff, which I'm okay with 'cause I like formulas okay, you know? But so this one was, like, if  $f(0)$  is 2, and  $f(1)$  is 2, what could the average value of  $f$  on the interval from 0 to 1 be? So I said none of the above, since 2 wasn't on the list, but if they don't tell you the formula for what  $f$  is I really don't think there's any way to figure it out. This girl in my class said it was all of the above, but that's obviously dumb, since everybody knows in math there's only one right answer."

Help Biff out by explaining clearly what possible average values such functions might have.

8. [Briggs/Cochran, §5.5]

- a) A change of variables that can be interpreted geometrically is the scaling  $u = cx$ , where  $c$  is a real number. Prove that

$$\int_a^b f(cx) dx = \frac{1}{c} \int_{ac}^{bc} f(u) du.$$

- b) Draw a picture to illustrate this change of variables in the case where  $f(x) = \sin x$ ,  $a = 0$ ,  $b = \pi$ , and  $c = 1/2$ .

9. In the movie *Star Wars*, the Death Star is a sphere 140,000 meters in diameter. The trench Luke had to fly through was 20 meters deep and 40 meters wide, running around the equator of the sphere. Write an integral for the volume of the trench, that is, the volume of the region between the bottom of the trench and what would have been the surface of the sphere if there were no trench.

10. A well is shaped like a cylinder with a vertical axis with height  $h$  and radius  $r$ , both in feet. If all of the water (with a density of  $62.5 \text{ lbs/ft}^3$ ) in the well is to be pumped to a point  $d$  feet above the top of the well, write an integral for the total amount of work required.

Extra Credit (5 points possible):

The volume of a sphere with radius  $r$  is  $\frac{4}{3} \pi r^3$ . Find a value  $h$  such that the volume from the bottom of the sphere up  $h$  units will be one-quarter the volume of the whole sphere.